Operations Research

Master in Mechanical Engineering

Introductive lecture

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Via Ariosto 25

Operations Research
Lab. of Operations Research

- ECTS:
  - Operations Research 6 cfu
  - Lab. of Operations Research 3 cfu

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- Teaching material and info available on the web
  `http://www.dis.uniroma1.it/~palagi/`
- Please sign in the Google group

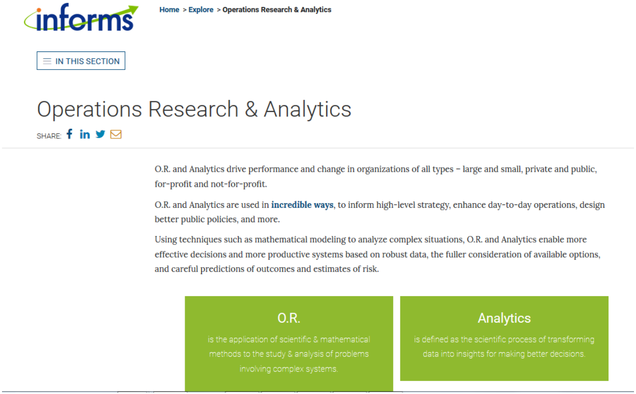
Exam & Grading

- Operations Research (6 cfu)
  - Written exam(maximum grade is 31 = 30 cum laude); at least greater than 18 to go to the oral part;
  - Oral exam (the maximum grade is 30): compulsory with written over 24;
  - In case of oral, the final grade is the arithmetic average of the two parts. The passing grades are from 18 to 30 cum laude
- Operations Research Laboratory (3 cfu)
  - Homework during the class.
  - HW receive a grade up to 30. The passing grades are from 18 to 30.
  - Whenever the Lab. OR grade is greater than 26/30, this grade can enter the arithmetic average of the OR exam

What's about O.R. Operations Research (USA) - Operational Research (UK)

The terms "decision science", "management science" and also "analytics" are sometimes used as synonyms for O.R.

See e.g. https://www.informs.org/



informs

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IN THIS SECTION

Operations Research & Analytics

SHARE:

O.R. and Analytics drive performance and change in organizations of all types – large and small, private and public, for-profit and not-for-profit.

O.R. and Analytics are used in **incredible ways**, to inform high-level strategy, enhance day-to-day operations, design better public policies, and more.

Using techniques such as mathematical modeling to analyze complex situations, O.R. and Analytics enable more effective decisions and more productive systems based on robust data, the fuller consideration of available options, and careful predictions of outcomes and estimates of risk.

O.R. is the application of scientific & mathematical methods to the study & analysis of problems involving complex systems.

Analytics is defined as the scientific process of transforming data into insights for making better decisions.

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The origin

- The term OR was initially applied in Britain just prior to World War II to denote the efforts of military planners.
- History is not clear and different people have different views of the same event. We report a brief version of Prof. J. E. Beasley.
  http://people.brunel.ac.uk/ mastjjb/jeb/jeb.html.

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- Between 1935 and 1937 UK developed a control system for fighter aircraft based on the use of the RAdio Detection And Ranging.
- (1936) Royal Air Force (RAF) started pre-war radar experiments at the Bawdsey Research Station.

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Biggin Hill Experiment

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- (1938) Four additional radar stations had been installed along the coast BUT the aircraft location and control system did NOT improve neither in coverage nor in effectiveness.

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- There was the need to coordinate and correlate the, often conflicting, information received from the radar stations.
- The Superintendent of Bawdsey Research Station, A.P. Rowe proposed to start a research program into the operational - as opposed to the technical - aspects of the system.
- The term ”operational research” was coined.

Operational Research Section

- So, the military services began assembling scientists, engineers and mathematicians into groups to conduct research directly in support of high-level officers. Air Chief Marshal Sir Hugh Dowding was the chief of the OR team.

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- after (1941) RO was used on lot of problems such as
  1. organisation of flying maintenance and inspection
  2. comparison of aircraft type
  3. improvement of attack kill probability (the probability of attacking and killing a U-boat)
  4. etc

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- Among the researcher there was

  George B. Dantzig (1914 - 2005)

  http://news.stanford.edu/news/2005/may25/dantzigobit-052505.html

  who worked at the Pentagon as an expert on programming-planning methods using desk calculators. He was inspired by the Interindustry Input-Output Model of the American Economist of Wassily Leontief (Nobel Prize 1976) and posed for the first time in 1947 *linear programming problem* (LP) and a method to compute its solution *the simplex method*.
- (1948) Dantzig presented LP to a meeting of the Econometric Society in Wisconsin.

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Dantzing's memory

http://web.udl.es/usuaris/MatFDiE/OptiSim/2LPOperRes02.pdf " *I was a young unknown and I remember how frightened I was with the idea of presenting for the first time to such a distinguished audience the concept of linear programming. After my talk, ... a hand was raised. It was Hotellings. I must hasten to explain that Hotelling was fat. ..... This huge whale of a man stood up in the back of the room,..... He said:*

But we all know the world is nonlinear.

Having uttered this devastating criticism of my model, he majestically sat down. ...... Suddenly another hand in the audience was raised. It was von Neumann...who.... said: The speaker titled his talk linear programming and carefully stated his axioms. If you have an application that satisfies the axioms, well use it. If it does not, then dont, and he sat down.

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Starting with the RAND Corporation....

- (1948) Following the end of the war OR took a different course in the UK as opposed to in the USA. In the UK many of the distinguished OR workers returned to their original peacetime disciplines. In the USA OR spread to the universities so that systematic training in OR began. In 1948 project RAND started in the USA (research and development) (see http://www.rand.org/about/history/)

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- (1951) The Karush-Kuhn-Tucker optimality condition for Non Linear Programming
- (1954) Ragnar Frisch (the first Nobel prize in Economics) proposed an interior point methods for LP
- (1954) William Orchard-Hays of the Rand Corporation, wrote the first commercial software for solving linear programs

...to nowadays

OR was born in a pre-computer period, nevertheless the impact of using OR techniques were quite significant.
In the 21th century the increasing power and widespread availability of computers allow the use of high quantities of data and increases calculating capacity. OR supplies the analytic and solving tools.

"O.R. is the application of advanced analytical methods to help make better decisions on complex decision problems"

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Why one should use analytic tools ?

let's check on a simple example *s*taff assignment.

G.B. Dantzig - Linear Programming: *the story about it began: some legends, a little about historical significance, and comments about where its many mathematical programming extensions may be headed* in History of Mathematical programming - a collection of personal reminiscences, J.K. Lenstra, A.H.G. Rinnooy Kan and A. Schrijver eds., NorthHolland (1991).

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A company considers the problem of assigning 70 men to 70 jobs.
Of course

- each man must be assigned a job (there are 70 such)
- each job must be filled (also 70)

It is important to the company how to do assignment.

The assignment problem

The reasons why different assignment can have different values may be due to

- preference of each man that performs certain tasks more willingly than others or
- different working skills of each individual employee

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Suppose a value or benefit c_{ij} is known for each possible assignment of the i-th man to the j-th job.
The problem consists in comparing all the possible assignments of employees to activities and select the best one.

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Of course it depends on the point of view !

- To the company
  global point of view: “best one” may mean the one that allows to get the *m*aximum average gain in terms of *satisfaction/performance*

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We assume a global viewpoint when there is only one decision maker

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More formally, we have

- a finite set D of men $\{1, 2, \ldots, 70\}$,
- a finite set A of jobs $\{1, 2, \ldots, 70\}$,
- for each pair (man$\sharp i$, job$\sharp j$) a value or benefit $c_{ij} > 0$;

As an example, we can assume that

$$c_{ij} = \begin{cases} 1 & \text{if man}\sharp i \text{ likes job}\sharp j \\ < 1 & \text{otherwise} \end{cases}$$

"best one" = maximum average gain

The average gain can be expressed as the sum of the values c_{ij} corresponding to the assignment of man$\sharp i$ to job$\sharp j$.

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The ideal solution would be an assignment of value equal to 70, but this is not always possible (actually almost never).

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The 3 dimensional assignment

The assignment problem has been proposed for the first time n 1950 on the journal (Psychometrika, 15(3), (1950) "The problem of classification of personnel").

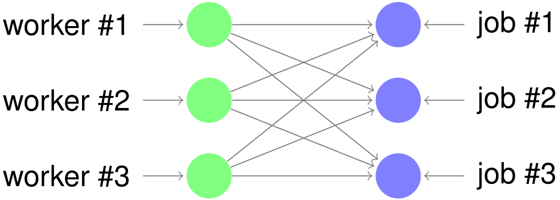
Given: A set of k job categories with N vacancies to be filled ($N \geqq k$), and N individuals to be used in filling them.

Required: To assign the individuals to the jobs in such a way that the average success* of all the individuals in all the jobs to which they are assigned will be a maximum.

TABLE 1
Aptitudes of Three Individuals for Three Jobs

Individual	*Job A*	*Job B*	*Job C*
I	55	60	65
II	50	50	55
III	45	50	45

The 3 dimensional assignment



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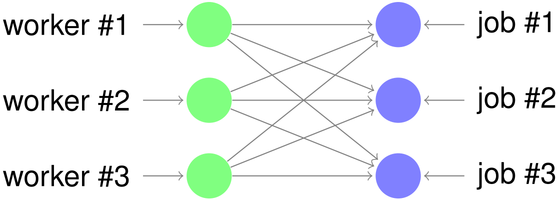

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worker♯1	0,9	0,8	1
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worker♯3	0,8	0,4	1

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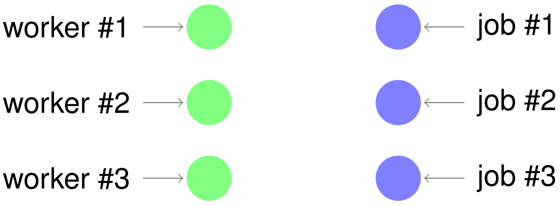
worker #1
worker #2
worker #3
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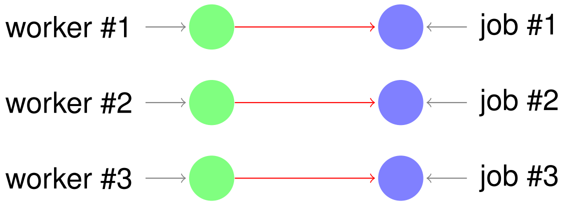
job #2
job #3



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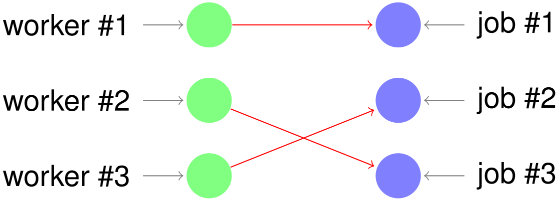

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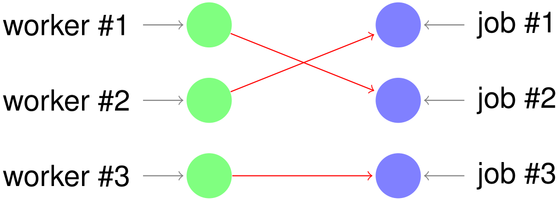

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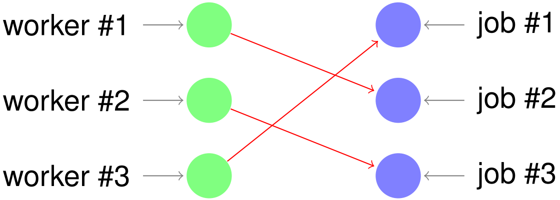

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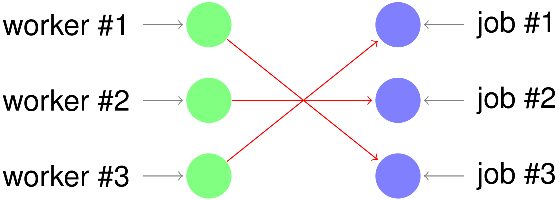

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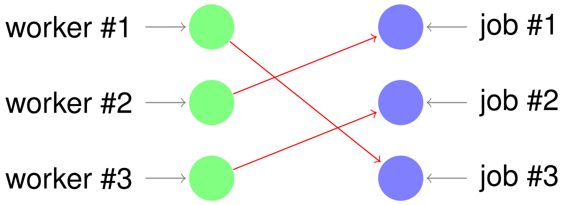

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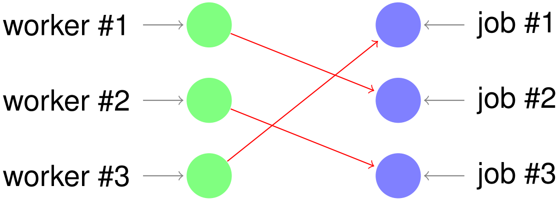

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Let's evaluate all the possible assignments.

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$\{(1,2),(2,1),(3,3)\}, c_{12} + c_{21} + c_{33} = 2,5$

$\{(1,2),(2,3),(3,1)\}, c_{12} + c_{23} + c_{31}$=2,6

$\{(1,3),(2,2),(3,1)\},, c_{13} + c_{22} + c_{31} = 2,4$

$\{(1,3),(2,1),(3,2)\}, c_{13} + c_{21} + c_{32} = 2,3$

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Complete enumeration

	job♯1	job♯2	job♯3
worker♯1	0,9	0,8	1
worker♯2	0,7	0,5	1
worker♯3	0,8	0,4	1

	job♯1	job♯2	job♯3
worker♯1	c_{11}	c_{12}	c_{13}
worker♯2	c_{21}	c_{22}	c_{23}
worker♯3	c_{31}	c_{32}	c_{33}

Let's evaluate all the possible assignments.

$\{(1,1),(2,2),(3,3)\}, c_{11} + c_{22} + c_{33} = 0,9 + 0,5 + 1 = 2,4$

$\{(1,1),(2,3),(3,2)\}, c_{11} + c_{23} + c_{32} = 0,9 + 1 + 0,4 = 2,3$

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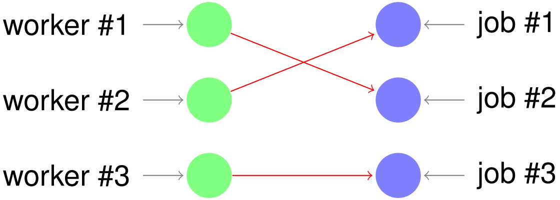
Complete enumeration

The best choice among all those satisfying restrictions is $\{(1, 2), (2, 3), (3, 1)\}$ of value 2,6



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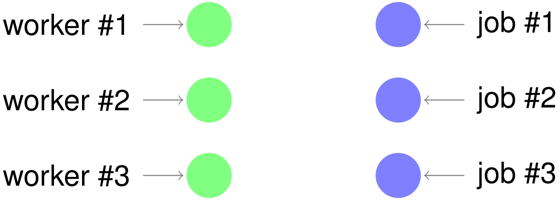

Attention ! there are combination with a value better than 2,6 but they do not satisfy the restrictions of covering all the jobs.

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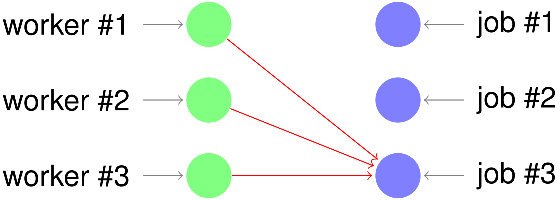

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e.g: $\{(1,3),(2,3),(3,3)\}$ has value 3

Complete enumeration

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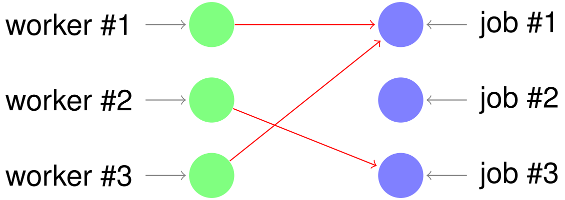

Attention ! there are combination with a value better than 2,6 but they do not satisfy the restrictions of covering all the jobs.
e.g: $\{(1,3),(2,3),(3,3)\}$ has value 3
or $\{(1,1),(2,3),(3,1)\}$ with value 2,7 !
Although these choices have higher return, they do not solve the required problem.

Complete enumeration

How many are the choices?

Complete enumeration

How many are the choices?

number of workers	choices
3	$6 = 3 \cdot 2$

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Complete enumeration

How many are the choices?

number of workers	choices
3	$6 = 3 \cdot 2$
4	$24 = 4 \cdot 3 \cdot 2$

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Complete enumeration

How many are the choices?

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3	$6 = 3 \cdot 2$
4	$24 = 4 \cdot 3 \cdot 2$
5	$120 = 5 \cdot 4 \cdot 3 \cdot 2$

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Complete enumeration

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5	$120 = 5 \cdot 4 \cdot 3 \cdot 2$
$\vdots$	$\vdots$
n	$n!$

When the number of workers/jobs is 70
all possible assignments are 70!

For each of the 70! one needs to get the value of the choice and then select the best one.

Complete enumeration

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When the number of workers/jobs is 70
all possible assignments are 70!

For each of the 70! one needs to get the value of the choice and then select the best one. How “big” is 70! ?

Veeeeeeery BIG

n	2	4	6	10	15	20	60	70
n !	2	24	720	3628800	1307674368000	2432902008176640000	-	-
order of	1	10	10^2	10^6	$\sim 10^{12}$	$\sim 10^{18}$	$\sim 10^{81}$	$\sim 10^{100}$

the possible choices are a huge number, larger than 10^{100}.

Can we do with the current technology ?

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Can we do with the current technology ?

In November 2016, the world's fastest supercomputer becomes the Sunway TaihuLight (National Supercomputing Center in Wuxi) with a LINPACK benchmark rating of 93 peta(10^{15})flops. This is nearly three times as fast as the previous holder of the record, the Tianhe-2, which ran at 34 petaflops. In computing, FLOPS (FLoating-point Operations Per Second) is a measure of computer performance, it is a count of the floating point operations carried out by a given algorithm or computer program in one second.

How long will it take to such a computer to examine all the 70! choices ?

Assuming each evaluation corresponds to one flops (that's not !) we can roughly state

In one year we have (365*24*3600=)31536000$\sim 3 \cdot 10^7$ seconds,
so that Sunway TaihuLight can elaborate

$$\sim 3 \cdot 10^7 \cdot 93 \cdot 10^{15} =\sim 10^{25}$$

Floating Point Operations.
Hence Sunway TaihuLight should work about $10^{100}/10^{25} = 10^{75}$ years to enumerate all the choices.

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When should have it started working ?

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The age of the universe is about 13 billion years ($13 \cdot 10^9$) !!!!!

The answer is NO !
Even if it would have worked from the time of the Big Bang.

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The answer is NO !
Even if it would have worked from the time of the Big Bang.

"If, there were 10^{40} *Earths circling the sun each filled solid with nanosecond speed computers all programmed in parallel from the time of the big bang until the Sun grows cold, then perhaps the answer might be, YES." - G. Dantzig*

Why use analytical tools ?

- In most cases it is unreasonable to check all the possible solutions to get the best one.

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What's O.R: ?

OR can be defined as as a quantitative approach to the solution of decision problems arising in organized structured systems.

- Interdisciplinarity OR joins methods and techniques from different fields (linear algebra, statistics, queue theory, system and control theory, game theory etc.)
- Unitary and overall view of a complex system. Most of real system involve many perspectives so that the analysis of mutually interacting is essential.
- Modelling and optimization approach in contrast to *"ground rule" approach.* Relying on "common sense" rules issued by a leader with experience and mature judgment in the field is no longer sufficient to facing the growing organizational complexity production systems and service.

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O.R. gives executives the power to make more effective decisions and build more productive systems

From INFORMS website: FAQ1

What are O.R. and analytics?

https://www.informs.org/Resource-Center/INFORMS-Student-Union/Career-FAQs#Q1 80% Cerca

Q1. What are O.R. and analytics?

The field of **operations research** (O.R.) began in the 1940s as mathematicians developed techniques for practical problem solving. Today, O.R. is the application of advanced analytical methods to help make better decisions. Closely connected to O.R., **analytics** is the scientific process of transforming data into insight for making better decisions. Both offer exciting ways to apply math methods to real-world situations and everyday decision making.

- Websites like LinkedIn and Google use O.R. and analytics to make behind-the-scenes connections among personal profiles. Tech companies like IBM, Intel, and HP use O.R. and analytics to strengthen management, improve the way they manufacture, and reduce resource use – all to create less expensive computers and smartphones.
- The U.S. Army uses O.R. and analytics to plan the delivery of supplies and to fight terrorism.
- Airlines use O.R. and analytics to schedule your flight crew and your flight, to protect passengers, and to set prices.
- Humanitarian relief agencies use O.R. and analytics to plan for disasters and, when disaster strikes, to rush food and medicine to those in need.
- Organizations of every type use O.R. and analytics to attack problems with lots of choices and even more data to come up with optimal solutions.
- Other areas:
  - Healthcare
  - Sports
  - Social media
  - Environmental work
  - Product development
  - Advertising/marketing
  - Decision making
  - **Logistics**

What does “model” means ?

The word *model* is usually used to denote a representation, generally in reduced size, to show the properties of some real object.

- prototype of an aircraft or a car,

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What does “model” means ?

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- prototype of an aircraft or a car,
- *abstract models* i.e *mathematical models* that uses mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling.

OR uses mathematical models that extracts the essential aspects and which summarizes the interrelationships between the different aspects of the phenomenon being studied.

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OR models

Why a mathematical model ?

- force to an analysis to grasp the essential and significant aspects of a problem
- Analytical deduction of structural properties otherwise not evident
- possibility of solution when the choices are ’so many’ that enumeration is impossible in practice
- possibility of evaluating “off-line” the effect of the choices

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OR models can be classified using different paradigms

- *stochastic model* which are models involving random variation
- *deterministic models*;
- *static models* when interactions among quantities are immediate
- *dynamic models* when interactions among quantities are time depedent.

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Mathematical models are usually composed of relationships and variables.
Relationships can be described by operators, such as algebraic operators, functions, differential operators, etc.
Variables are abstractions of system parameters of interest, that can be quantified.
Models tackled in this course are *Mathematical Programming* problems.

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OR devotes to development of mathematical methods (solution algorithms) to obtain the optimal solution (or a good approximation) of the problem

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What does algorithm mean ?

It is a self-contained step-by-step set of operations to be performed to obtain in finite time the solution of a given problem.

The oldest and still most famous algorithm in OR is the simplex method which solves a linear programming problem and it can be used as a tools also for different class of problems.

Principles of Mathematical Modeling

- **Description and analisys of the problem**
- **Model construction**
- **Model Analysis**
- **Selection of “good” solutions**
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Principles of Mathematical Modeling

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- **Model Validation**
  - verify wether the model is consistent with its principles and assumptions

The mathematical model of assignment

- Choice of the variables

$$x_{ij} = \begin{cases} 1 & \text{if worker } i \text{ is assigned to job } j \\ 0 & \text{otherwise} \end{cases} \quad i, j = 1, \dots, n$$

- Choice of the objective function

$$\text{maximize} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

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The mathematical model - 2

Choice of the constraints

- At each job is assigned exactly one worker

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for all } j = 1, \ldots, n$$

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The correct model

Just to start to understand what's coming on

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The correct model

Just to start to understand what's coming on

Mathematical modelling of the assignment problem:

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{70} \sum_{j=1}^{70} c_{ij} x_{ij} \\
& \sum_{i=1}^{70} x_{ij} = 1 \quad \text{for all } j = 1, \dots, 70 \\
& \sum_{j=1}^{70} x_{ij} = 1 \quad \text{for all } i = 1, \dots, 70 \\
& x_{ij} \in \{0, 1\} \quad i = 1, \dots, 70 \;\; j = 1, \dots, 70
\end{aligned}
$$

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Properties of the model

- It is a mathematical programming model

$$
\begin{aligned}
\max \quad & f(x) \\
& x \in S \\
& x_{ij} \in \{0,1\} \quad i = 1,\dots,n \;\; j = 1,\dots,n
\end{aligned}
$$

  with *linear* functions describing the objective and the constraints.

- Variables can assume only boolean variables (later on you will learn that the boolean restrction can be removed without effect on the solution)