SAPIENZA
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Operations research

ILP model for the problem

Optimize your Investments

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VIDEO

In the video (in italian) you find a brief description of the problem

DESCRIPTION

investiments	A	B	C	D	E	F	G	H	I	L
Pay	24	30	18	30	56	15	22	21	26	15
Obtain	38	50	27	51	86	22	33	35	46	23
Which do you choose ?	☐	☐	x	☐	x	☐	☐	☐	☐	☐

If you choose C and E,

You pay 18+56=74

Surplus = 100-74=26

You obtain 27+86=113

Budget 100

Overall you get 113+26=139 Keuro

Enumeration ?

$$x_i = \begin{cases} 0 & if\ investment\ i\ is\ NOT\ selected \\ 1 & if\ investment\ i\ is\ selected \end{cases}$$

investments												
A	B	C	D	E	F	G	H	I	L	Income	surplus	revenue
0	0	0	0	0	0	0	0	0	0	0	100	100
1	0	0	0	0	0	0	0	0	0	38	76	114
0	1	0	0	0	0	0	0	0	0	50	70	120
0	0	1	0	0	0	0	0	0	0	27	82	109
0	0	0	1	0	0	0	0	0	0	51	70	121
				1						86	44	130
					1					22	85	107
						1				33	78	111
							1			35	79	114
								1		46	74	120
									1	23	85	108
1	1	0								88	46	134
1		1								65	58	123
1			1							89	46	135
1				1						120	20	140
1					1					60	61	121

How many are they ?

Possible combinations of values {0,1} on the 10 investments

$2^{10} = 1024$ in general 2^n

- Enumeraton is possible only for small values of n
- Solutions obtained by enumeration depend from the specific values of income, costs and budget. If these values change, solution must be evaluated again

- It does not allow the use of *algorithms*

Decisional choices

Si tratta di individuare quali sono le leve decisionali su cui possiamo agire. In generale indicano la risposta che dobbiamo fornire al decisore.

$$x_i = \begin{cases} 0 \quad if\ investment\ i\ is\ NOT\ selected \\ 1 if\ investment\ i\ is\ selected \end{cases}$$

As an example, the choice B,C,D,H is identified as follows:

	A	B	C	D	E	F	G	H	I	L
INVESTMENT	0	1	1	1	0	0		1		0

$$x \in \{0,1\}^n$$

Costs

If we choose only investment B, the cost is 30

let us denote by c_i the cost values, namely

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
24	30	18	30	56	15	22	21	26	15

We can express the cost in analytic way by using the defintion of x_i

$$c_1x_1 + c_2x_2 + \cdots + c_ix_i \ + \cdots + c_{10}x_{10} = \sum_{i=1}^{10} c_ix_i$$

The budget constraint

$$\sum_{i=1}^{10} c_i x_i \leq budget$$

Objective

Our objective is to *maximize* the cash = revenue + remaining cash

Revenue $r_1 x_1 + r_2 x_2 + \cdots + r_i x_i + \cdots + r_{10} x_{10} = \sum_{i=1}^{10} r_i x_i$

$$\max \sum_{i=1}^{10} r_i x_i + (b - \sum_{i=1}^{10} c_i x_i)$$

The full model

$$\max \sum_{i=1}^{10} r_i x_i + (b - \sum_{i=1}^{10} c_i x_i)$$

$$\sum_{i=1}^{10} c_i x_i \leq b$$

$$x \in \{0,1\}^n$$

Integer Linear Programming Problem