A blending problem

A OIL is manufactured by refining row oils and blending them together.

The raw oils are available in two main categories: vegetables (VEG1, VEG2) and Non-vegetables oils (OIL1,OIL2,OIL3)

Each of the main ingredients has a cost (€per tons): VEG1, VEG2 cost respectively 110 € and 120 €, and OIL1, OIL2, OIL3 cost respectively 130 €, 110 € and 115 €.

The final product is sold at 150 € per tons.

The quality of an oil is measured in hardness. The hardness of the components are reported in the table

	VEG1	VEG2	OIL1	OIL2	OIL3
hardness	8,8	6,1	2	4,2	5

There are quality restriction on the hardness of the final oil product. In percentage the hardness of the final oil must lie between 3 and 6. It is assumed that hardness of the product behaves linearly with the hardness of the components.

You need to construct an LP model for the managing the production so to maximize the profit.

The mathematical model

– *Parameters.*

- $m = 5$ number of raw oils
- p profit PER TONS
- c_i cost per tons of each raw oil.
- h_i hardness of each raw oil
- $[H^{\min}, H^{\max}]$ min e max hardness of the final oil
- T target value of the production

We observe that we need to fix the value T although not specified in the text because otherwise the problem would be unbounded, being no restriction on the production. Indeed we would like to determine the proportion of ingredients that respect the constraints and maximize the objective function.

– *Decision variables.* x_i $i = 1, \ldots, m$ tons of products of type i

– *Objective function.* maximize profit

$$\max_x \; p \sum_{j=1}^{m} x_j - \sum_{j=1}^{m} c_j x_j$$

– *Constraints.*

- hardness constraints

  The hardness is given by

$$\text{hardness} = \frac{\sum_{i=1}^{n} h_i x_i}{\sum_{i=1}^{n} x_i}$$

  that must stays in the interval $[H^{\min}, H^{\max}]$. Since we need linear constraints, we must write the constraints as

$$\sum_{i=1}^{n} h_i x_i \geq H^{\min} \sum_{i=1}^{n} x_i$$
$$\sum_{i=1}^{n} h_i x_i \leq H^{\max} \sum_{i=1}^{n} x_i$$

- target value of the production

$$\sum_{i=1}^{n} x_i = T$$

- non negativity $x \geq 0$

The full model is

$$\begin{aligned} \max \quad & p \sum_{j=1}^{m} x_j - \sum_{j=1}^{m} c_j x_j \\ & \sum_{i=1}^{n} h_i x_i - H^{\min} \sum_{i=1}^{n} x_i \geq 0 \\ & \sum_{i=1}^{n} h_i x_i - H^{\max} \sum_{i=1}^{n} x_i \leq 0 \\ & \sum_{i=1}^{n} x_i = T \\ & x_i \geq 0 \quad i = 1, \ldots, m \end{aligned}$$

Additional constraints

It is possible to impose additional constraints, e.g. a minimum level of usage of each of the oils. Additional parameters

- ℓ_i minimum amount of raw oils used in the blend

Additional constraint $x_i \leq \ell_i$