Operations Research Laboratory

Master in Mechanical Engineering

A blending problem with a convex piecewise function

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A blending problem

An oil company must fulfill four blending orders using three crude oils.

quality	request	order
at least 7	2000	1
at most 7.8	1500	2
between 7.0 and 8.0	2500	3
7.4	3000	4

Crude oil	1	2	3
revenue euro per barrel	15	17	20
availability, max barrel per day	4000	3000	5000
quality	6.8	7.4	8.1

Decision variables for blending

quality	request	order			
at least 7	2000	1	x_{11}	x_{12}	x_{13}
at most 7.8	1500	2	x_{21}	x_{22}	x_{23}
between 7.0 and 8.0	2500	3	x_{31}	x_{32}	x_{33}
7.4	3000	4	x_{41}	x_{42}	x_{43}
		Crude oil	1	2	3
		revenue euro per barrel	15	17	20
		availability, max barrel per day	4000	3000	5000
		quality	6.8	7.4	8.1

Constraints

Availability constraints

$$\sum_{i=1}^{4} x_{i1} \leq 4000$$

$$\sum_{j=1}^{4} x_{i2} \leq 3000$$

$$\sum_{i=1}^{4} x_{i3} \leq 5000$$

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Constraints

Demand constraints

$$\sum_{j=1}^{3} x_{1j} = 2000$$

$$\sum_{j=1}^{3} x_{2j} = 1500$$

$$\sum_{j=1}^{3} x_{3j} = 2500$$

$$\sum_{j=1}^{3} x_{4j} = 3000$$

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Constraints

Quality constraints

$$6.8x_{11} + 7.4x_{12} + 8.1x_{13} \geq 7(\sum_{j=1}^{3} x_{1j})$$

$$6.8x_{21} + 7.4x_{22} + 8.1x_{23} \leq 7.8(\sum_{j=1}^{3} x_{2j})$$

$$7(\sum_{j=1}^{3} x_{3j}) \leq 6.8x_{31} + 7.4x_{32} + 8.1x_{33} \leq 8(\sum_{j=1}^{3} x_{3j})$$

$$6.8x_{41} + 7.4x_{42} + 8.1x_{43} = 7.4(\sum_{j=1}^{3} x_{4j})$$

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The objective function

Revenue

$$R = 15\sum_{i=1}^{4} x_{i1} + 17\sum_{i=1}^{4} x_{i2} + 20\sum_{i=1}^{4} x_{i3} = \sum_{j=1}^{3} r_j \sum_{i=1}^{4} x_{ij}.$$

Quality of order 3

$$Q_3 = \frac{6.8x_{31} + 7.4x_{32} + 8.1x_{33}}{\sum_{j=1}^{3} x_{3j}} = \frac{6.8x_{31} + 7.4x_{32} + 8.1x_{33}}{2500}$$

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The objective function

$$\max \; R - P|Q_3 - 7.5|$$

$$\max \sum_{j=1}^{3} r_j \sum_{i=1}^{4} x_{ij} - P \left| \frac{6.8x_{31} + 7.4x_{32} + 8.1x_{33}}{2500} - 7.5 \right|$$

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