OPERATIONS RESEARCH

SIMULATION EXAM December 23, 2019

IMPORTANT: READ CAREFULLY

The grade on the written exam is valid at most for three months.

Check the score of each exercise.

Please note that YES NO answers are NOT valid if you do not give an explanation.

SURNAME **NAME**

I **authorize** the publication (paper and electronic) of the results obtained in the examination in accordance with the Italian Law 675/96 and subsequent amendments

SIGNATURE

GRADE

I solved the following exercise

1. ☐ **(Score 8)**
2. ☐ **(Score 4)**
3. ☐ **(Score 7)**
4. ☐ **(Score 6)**
5. ☐ **(Score 6)**

ORAL PART: questions

GRADE

Exercise 1. **(Score 8)** Consider the following nonlinear programming problem

$$\min_{x\in I\!R^3} \quad 3x_1^2 + 2x_2^2 + \frac{5}{2}x_3^2 + x_1x_3 - 2x_2x_3 + 3x_1 - 4x_2 + x_3$$

$$\begin{array}{l} 3x_1 + 2x_2 + x_3 = 2 \\ x_1 + x_2 + x_3 \geq -1 \\ -2x_1 + x_2 - x_3 \leq 3 \\ x_2 \geq 0 \end{array}$$

Unconstrained problem (score 2)

(i) **(score 1)** Consider the unconstrained problem (remove the constraints). Check if there exists any unconstrained stationary point and, in case, state which kind of point it is

(ii) **(score 1)** Consider the point $x^0 = (0,\ 0,\ 0)^T$ and write the first iteration of the gradient method with exact line search to obtain the new point x^1

Constrained problem (score 6)

(iii) **(score 1)** State if the problems is convex or strictly convex or none of the two.

(iv) **(score 2)** Consider the point $\widehat{x} = (1,\ 1,\ -3)^T$ and write the KKT conditions in $\widehat{x}$. Evaluate the multipliers. Are the KKT conditions satisfied ?

(v) **(score 1)** Write the system to get a feasible and descent direction in $\widehat{x}$. Does a solution of the system (namely a feasible and descent direction) exist?

(vi) **(score 2)** Consider the point $x^0 = (1,\ 1,\ -3)^T$ and write the linear programming problem (in the variables x) to find the direction d^0 at the first iteration of the conditional gradient (Frank-Wolfe) method. Assume that the optimal solution of this Linear problem is $x^{0*} = (0,\ 0,\ 2)^T$, write the direction d^0 and find the value $t^{\max} = \min\{t_{FEAS}, t^*\}$ where t^* is obtained by an exact Line Search

Exercise 2. **(Score 4)** Consider the following Linear programming problem

$$\min \quad 6x_1 + 8x_2 - 15x_3$$

$$\begin{array}{l} 3x_1 + 2x_2 + x_3 = 2 \\ x_1 + x_2 + x_3 \geq -1 \\ -2x_1 + x_2 - x_3 \leq 3 \\ x_1, x_2, x_3 \geq 0 \end{array}$$

Consider the point $\widehat{x} = \left(\frac{1}{3},\ \frac{1}{2},\ 0\right)^T$.

(i) **(score 0,5)** Is the point $\widehat{x}$ a vertex ?

(ii) **(score 1,5)** Find a feasible direction along which it is possible to move from $\widehat{x}$ and find an additional active constraint besides those already active in $\widehat{x}$. Check if the direction is a descent one.

Find the stepsize $t^{\max}$ and the corresponding new point y. Is the point y a vertex ?

(iii) **(score 0,5)** Write the problem in the standard form for the simplex method.

(iv) **(score 1)** Write a BFS for the problems at point (iii) and specify the corresponding matrices B and N.

(v) **(score 0,5)** Write the auxiliary problem in (x, α) for the Phase I of the simplex method. Write any BFS for the auxiliary problem (only the values of the variables (x, α)).

Exercise 3. **(Score 7)** Consider the following Linear programming problem

$$\begin{aligned} \min \quad & 6x_1 + 8x_2 - 15x_3 \\ & 3x_1 + 2x_2 + x_3 = 2 \\ & -2x_1 + x_2 - x_3 \leq 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(NB It is not the same of the Exercise 2: one constraint is missing)

(i) **(score 1.5)** Write the dual problem

(ii) **(score 2)** Solve graphically the dual: plot the feasible region, the level lines of the objective function, identify graphically the solution and find its value.

(iii) **(score 2)** Using duality theory, state if the primal problem has an optimal solution x^* and in the affirmative case find it.

(iv) **(score 0.5)** State how the value of the optimal solution change if the rhs of the first constraint change from 2 to $2 + \varepsilon$ with ε sufficiently small.

(v) **(score 1)** Consider the BFS corresponding to the optimal vertex x^* (namely find the values of the slack and/or surplus variables associated to the standard form of the given problem). Find the values of the reduced costs γ

Exercise 4. **(Score 6)** Consider the following integer knapsack problem

$$\begin{aligned} \max \quad & 3x_1 + 8x_2 + \frac{3}{2}x_3 + x_4 \\ & 2x_1 + x_2 + 5x_3 + \frac{3}{2}x_4 \leq \frac{15}{2} \\ & x \in \{0, 1\} \end{aligned}$$

Solve the problem.

Exercise 5. **(Score 6)** Under normal working conditions a factory can produce up to 100 units of a certain product in each of four consecutive time periods at costs which vary from period to period as shown in the table below.

Additional units can be produced by overtime working in each of the four periods. The maximum quantity and costs of overtime production are shown in the table below, together with the forecast demands for the product in each of the four time periods.

Time Period	Demand (units)	Normal Production Costs (£K/unit)	Overtime Production Capacity (units)	Overtime Production Cost (£K/unit)
1	130	6	60	8
2	80	4	65	6
3	125	8	70	10
4	195	9	60	11

It is possible to hold up to 70 units of product in store from one period to the next at a cost of £1.5K per unit per period.

It is required to determine the production (both normal and overtime) and storage schedule which will meet the stated demands over the four time periods at minimum cost given that at the start of period 1 we have 15 units in stock.

Formulate this problem as an LP.