OPERATIONS RESEARCH

EXAM OCTOBER 26, 2019

IMPORTANT: READ CAREFULLY

The grade on the written exam is valid at most for three months.

Check the score of each exercise.

Please note that YES NO answers are NOT valid if you do not give an explanation.

SURNAME **NAME**

I **authorize** the publication (paper and electronic) of the results obtained in the examination in accordance with the Italian Law 675/96 and subsequent amendments

SIGNATURE

GRADE

I solved the following exercise

1. ☐ **(Score 7)**
2. ☐ **(Score 5)**
3. ☐ **(Score 8)**
4. ☐ **(Score 6)**
5. ☐ **(Score 6)**

ORAL PART: questions

GRADE

Exercise 1. **(Score 7)** Consider the following nonlinear programming problem

$$\begin{aligned} \min_{x \in I\!R^3} \quad & x_1^2 + \frac{3}{2}x_2^2 + x_3^2 - x_1x_3 - x_1x_2 + x_1 - 2x_2 - 3x_3 \\ & -3x_1 + 2x_2 - x_3 = 3 \\ & 2x_1 - 2x_2 + 3x_3 \le 4 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Unconstrained problem **(score 2)** Consider the unconstrained problem (remove the constraints)

(a) **(score 0.5)** Is the unconstrained problem convex or strictly convex ?

(b) **(score 1)** Find the stationary points of the unconstrained problem, if any. Which kind of point are they ? (local/global/saddle)

(c) **(score 1)** Consider the point $x^0 = (1,\ 3,\ 0)^T$ and write the first iteration of the gradient method with exact line search to obtain the new point x^1

Constrained problem **(score 5)**

(a) **(score 0.5)** State if the feasible region is convex.

(b) **(score 2)** Consider the point $\widehat{x} = (1,\ 3,\ 0)^T$ and write the KKT conditions in $\widehat{x}$. Evaluate the multipliers. Are the KKT conditions satisfied ?

(c) **(score 1)** Write the system to get a feasible and descent direction in $\widehat{x}$. Does a solution of the system (namely a feasible and descent direction) exist?

(d) **(score 1)** Consider the point $x^0 = (1,\ 3,\ 0)^T$ and write the linear problem to find the direction of the conditional gradient (Frank-Wolfe) algorithm.

Exercise 2. **(Score 4)** Consider the following Linear programming problem

$$\begin{aligned} \min \quad & x_1 + x_2 - 2x_3 \\ & -3x_1 + 2x_2 - x_3 = 3 \\ & 2x_1 - 2x_2 + 3x_3 \le 4 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

(i) **(score 1,5)** Find a a feasible direction along which it is possible to move from $\widehat{x} = (1,\ 3,\ 0)^T$ finding an additional active constraint. Find the stepsize $t^{\max}$ and the corresponding new point y. Is the direction also a descent one?

(ii) **(score 0,5)** Write the problem in the standard form for the simplex method.

(iii) **(score 1)** Check if the following point are vertex or not and justify the answer.

$$\begin{pmatrix} \frac{1}{2} \\ 3 \\ \frac{3}{2} \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ \frac{3}{2} \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ \frac{13}{4} \\ \frac{7}{2} \end{pmatrix}$$

(iv) **(score 1)** Write a BFS for the problem at (ii) with the corresponding matrices B and N.

Exercise 3. **(Score 8)** Consider the following Linear programming problem

$$\begin{aligned} \min \quad & x_1 + x_2 - 2x_3 \\ & -3x_1 + 2x_2 - x_3 = 3 \\ & 2x_1 - 2x_2 + 3x_3 \leq 4 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

- **(i) (score 1.5)** Write the dual problem
- **(ii) (score 2)** Solve graphically the dual: plot the feasible region, the level lines of the objective function, identify graphically the solution and find its value.
- **(iii) (score 2)** Using duality theory, state if the primal problem has an optimal solution and in the affirmative case find it.
- **(iv) (score 1)** State how the value of the optimal solution change if the rhs of the second constraint change from 4 to $4 + \varepsilon$ with $\varepsilon > 0$ and sufficiently small.
- **(v) (score 1.5)** State which is the maximum value of $\varepsilon > 0$ for which the analysis in point (iv) holds.

Exercise 4. **(Score 6)** Consider the following integer linear programming problem

$$\begin{aligned} \max \quad & 40x_1 + 90x_2 + 100x_3 + 60x_4 + 15x_5 + 15x_6 + 10x_7 + x_8 \\ & 40x_1 + 20x_2 + 20x_3 + 30x_4 + 2x_5 + 30x_6 + 60x_7 + 10x_8 \leq 102. \end{aligned}$$

- **(i) (score 1,5)** Give a lower and an upper bound on the optimal value.
- **(ii) (score 1,5)** Write the two subproblems obtained by branching with respect to the fractional variable
- **(iii) (score 3)** Solve the two suproblems generated and state if they can be closed or not motivating your answer

Exercise 5. **(Score 6)** A canning company[1] operates two canning plants A and B. The three growers[2] S1, S2, S3 are willing to supply fresh fruits in the following maximum amounts and costs:

grower	amount	cost
S1	200 tonnes	£11/tonne
S2	310 tonnes	£10/tonne
S3	420 tonnes	£9/tonne

Shipping costs in £ per tonne are:

	to	Plant A	Plant B
From:	S1	3	3.5
	S2	2	2.5
	S3	6	4

Plant capacities and labour costs are:

	Plant A	Plant B
Capacity	460 tonnes	560 tonnes
Labour cost	£26/tonne	£21/tonne

[1]industria conserviera
[2]coltivatori

The canned fruits are sold at £50/tonne to the distributors. The company can sell at this price all they can produce. The objective is to find the best mixture of the quantities supplied by the three growers to the two plants so that the company maximises its profits. Formulate the problem as a linear program.