OPERATIONS RESEARCH

Online EXAM June 15, 2020 on EXAM.NET

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TEST PART. (Score 5)

Each correct answer gives a score of 0,5.

You must answer AT LEAST 6 QUESTIONS CORRECTLY otherwise you get a penalty of -1

Consider the following optimization problem

$$\begin{array}{ll} \min & f(x_1, x_2, x_3) \\ & 6x_1 + 2x_2 + 6x_3 \geq 3 \\ & 2x_1 - x_2 + 3x_3 = 1 \\ & 3x_1 - 4x_2 + x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Answer True (T) or False (F) to the following questions:

1. The feasible region of problem (P) is a polyhedron
2. Problem (P) is a Linear Programing Problem
3. In the point $\tilde{x} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \dfrac{1}{2} \end{pmatrix}$ there are two active constraints.
4. The feasible region, if not empty, always admits a vertex
5. The point $\tilde{x} = \begin{pmatrix} \dfrac{1}{2} \\ 0 \\ 0 \end{pmatrix}$ is a vertex.
6. When the function f is concave, global minimizers, if any, lie in the interior of the feasible region
7. When the function $f(x)$ is convex, Problem (P) is a convex optimization Problem
8. Any point $\hat{x}$ satisfying $\nabla f(\hat{x}) = 0$ is a candidate to be a local constrained minimizer
9. If the function f is linear, problem (P) is a concave optimization problem
10. KKT conditions can be applied only when the function is convex

EXERCISE PART

Exercise 1. **(Score 8)** Consider the following problem

$$\begin{aligned} \min_{x\in \mathbb{R}^3} \quad & \frac{1}{2}x_1^2 + x_2^2 - 2x_3^2 + 2x_1x_3 - x_2x_3 - 2x_1 - 3x_3 \\ & 6x_1 + 2x_2 + 6x_3 \geq 3 \\ & 2x_1 - x_2 + 3x_3 = 1 \\ & 3x_1 - 4x_2 + x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(i) **(score 1)** State if the problems is convex or strictly convex (motivate your answer)

(ii) **(score 2.5)** Consider the point $\tilde{x} = \left(0,\ \frac{1}{2},\ \frac{1}{2}\right)^T$ and write the KKT conditions in $\widehat{x}$.
Write the values of the multipliers.
Are the KKT conditions satisfied ?

(iii) **(score 2)** Consider the **Unconstrained** problem (remove the constraints). Write the first iteration of the gradient method with exact line search starting from $\tilde{x} = \left(0,\ \frac{1}{2},\ \frac{1}{2}\right)^T$.
Write the new point x^1.

(iv) **(score 2.5)** Given the point $\tilde{x} = \left(0,\ \frac{1}{2},\ \frac{1}{2}\right)^T$, find a feasible direction $\bar{d}$ along which it is possible to move from $\widehat{x}$ and find an additional active constraint besides those already active in $\widehat{x}$.
Check if the direction is a descent one.
Find the stepsize $t^{\max}$.
Write the new point $y = x + t^{\max}\bar{d}$.

Exercise 2. **(Score 8)** Consider the following Linear programming problem

$$\begin{aligned} \min_{x\in \mathbb{R}^3} \quad & 9x_1 + 16x_2 + 12x_3 \\ & 6x_1 + 2x_2 + 6x_3 \geq 3 \\ & 3x_1 - 4x_2 + x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(i) **(score 1,5)** Write the dual problem.

(ii) **(score 2)** Solve graphically the dual problem: plot the feasible region, the level lines of the objective function, identify graphically the solution (you can use the tool Desmos on Exam.net and attach the figure obtained).
Report the optimal solution and the value of the objective function.

(iii) **(score 2)** Find, if it exists, a solution of the primal problem. Report the optimal dual solution and the value of the objective function.

(iii) **(score 1)** State how the value of the optimal solution change if the rhs of the first constraint change from 3 to $3 + \varepsilon$ with $\varepsilon > 0$ and sufficiently small.

(iv) **(score 1,5)** Write the primal problem in the standard form for the simplex methods by adding additional variables and find a BFS.

Exercise 4. (Score 5) Consider the following integer linear programming problem

$$\begin{aligned} \max \quad & 3x_1 - 2x_2 \\ & 6x_1 - 3x_2 \leq 9 \\ & 2x_1 + 4x_2 \leq 16 \\ & 6x_1 - x_2 \leq 12 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \quad \text{integer} \end{aligned}$$

Solve the ILP problem by the Branch-and-Bound method using the graphical solution for the LP relaxation. (you can use the tool Desmos on Exam.net and attach the figure obtained, if needed).

Report the optimal integer solution and the value of the objective function.

Exercise 3. (Score 5)

A company is planning its production schedule over the next six months (it is currently the end of month 2). The demand (in units) for its product over that timescale is as shown below:

Month	3	4	5	6	7	8
Demand	5000	6000	6500	4500	5500	9500
production cost per unit	15	15	15	13	14	12

The company currently has in stock: 1500 units which were produced in month 0.

The company can only produce up to 6000 units per month and the managing director has stated that stocks must be built up to help meet demand in months 5, 6, 7 and 8. Each unit produced cost that depends on the month and is reported in the table above (euro/unit) and the cost of holding stock is estimated to be £0.75 per unit per month.

The company wants a production plan for the next six months that avoids stockouts and that minimizes costs. Formulate their problem as a linear program.