OPERATIONS RESEARCH

Online EXAM Junly 24, 2020 on EXAM.NET

#Sapienzanonsiferma

TEST PART. (Score 5)

Each correct answer gives a score of 0,5.

You must answer AT LEAST 6 QUESTIONS CORRECTLY otherwise you get a penalty of -1

Consider the following optimization problem

$$\begin{array}{ll} \min & x_1 - 3x_2 + 2x_3 \\ & x_1^2 + 3x_2^2 + 2x_3^2 \le 2 \\ & 2x_1 - x_2 + 3x_3 = 1 \qquad (P) \\ & 3x_1 - 4x_2 + x_3 \le 2 \end{array}$$

Answer True (T) or False (F) to the following questions:

1. The feasible region of problem (P) is a polyhedron
2. Problem (P) is a Linear Programming Problem
3. The objective function is convex
4. In the point $\tilde{x} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \dfrac{1}{2} \end{pmatrix}$ there are two active constraints.
5. The optimization problem is convex
6. Global minimizers, if any, lie on the boundary of the feasible region
7. Any point $\hat{x}$ satisfying $\nabla f(\hat{x}) = 0$ is a candidate to be a local constrained minimizer
8. The gradient ∇f of the objective function of problem (P) never vanishes
9. KKT conditions can be applied to an optimization problem only when the function is convex
10. The dual problem of a Linear Programming Problem, is a liner programming problem itself

EXERCISE PART

Exercise 1. **(Score 8)** Consider the following problem

$$\begin{aligned} \min_{x\in I\!R^3} \quad & \frac{1}{2}x_1^2 + x_2^2 + x_3^2 + x_1x_3 - 2x_2x_3 + 3x_1x_2 + 2x_1 - 3x_2 + x_3 \\ & 6x_1 + 2x_2 + 6x_3 = 4 \\ & 2x_1 - x_2 + 3x_3 \geq 1 \\ & 3x_1 - 4x_2 + x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(i) **(score 1)** State if the problems is convex or strictly convex (motivate your answer)

(ii) **(score 2.5)** Consider the point $\tilde{x} = \left(0,\ \frac{1}{2},\ \frac{1}{2}\right)^T$ and write the KKT conditions in $\widehat{x}$.
Write the values of the multipliers.
Are the KKT conditions satisfied ?

(iii) **(score 2)** Consider the **Unconstrained** problem (remove the constraints). Write the first iteration of the gradient method with exact line search starting from $\tilde{x} = \left(0,\ \frac{1}{2},\ \frac{1}{2}\right)^T$.
Write the new point x^1.

(iv) **(score 2.5)** Consider the problem obtained by (P) removing the constraints $x_1, x_2, x_3 \geq 0$.
Given the point $\tilde{x} = \left(0,\ \frac{1}{2},\ \frac{1}{2}\right)^T$, find a feasible direction $\bar{d}$ along which it is possible to move from $\widehat{x}$ and find an additional active constraint besides those already active in $\widehat{x}$.
Check if the direction is a descent one.
Find the stepsize $t^{\max}$.
Write the new point $y = x + t^{\max}\bar{d}$.

Exercise 2. **(Score 8)** Consider the following Linear programming problem

$$\begin{aligned} \min_{x\in I\!R^3} \quad & 4x_1 + 8x_2 + 12x_3 \\ & 6x_1 + 2x_2 + 6x_3 = 4 \\ & 3x_1 - 4x_2 + x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(i) **(score 1,5)** Write the dual problem.

(ii) **(score 2)** Solve graphically the dual problem: plot the feasible region, the level lines of the objective function, identify graphically the solution (you can use the tool Desmos on Exam.net and attach the figure obtained).
Report the optimal solution and the value of the objective function.

(iii) **(score 2)** Find, if it exists, a solution of the primal problem. Report the optimal dual solution and the value of the objective function.

(iii) **(score 1)** State how the value of the optimal solution change if the rhs of the second constraint change from 1 to $1 + \varepsilon$ with $\varepsilon > 0$ and sufficiently small.

(iv) **(score 1,5)** Write the primal problem in the standard form for the simplex methods by adding additional variables and find a BFS.

Exercise 4. **(Score 5)** Consider the following integer linear programming problem

$$\begin{aligned}\max \quad & 4x_1 - x_2 \\ & 6x_1 - 3x_2 \leq 4 \\ & 2x_1 + 4x_2 \leq 8 \\ & 6x_1 - x_2 \leq 12 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \quad \text{integer}\end{aligned}$$

Solve the ILP problem by the Branch-and-Bound method using the graphical solution for the LP relaxation. (you can use the tool Desmos on Exam.net and attach the figure obtained, if needed).

Report the optimal integer solution and the value of the objective function.

Exercise 3. **(Score 5)**

A perfume firm produces two new items by mixing three essences: rose, lily and violet. For each decaliter of perfume one, it is necessary to use 1.5 liters of rose, 1 liter of lily and 0.3 liters of violet. For each decaliter of perfume two, it is necessary to use 1 liter of rose, 1 liter of lily and 0.5 liters of violet. 27, 21 and 9 liters of rose, lily and violet (respectively) are available in stock. The company makes a profit of 130 euros for each decaliter of perfume one sold, and a profit of 100 euros for each decaliter of perfume two sold. The problem is to determine the optimal amount of the two perfumes that should be produced.