OPERATIONS RESEARCH

EXAM January 16, 2020

IMPORTANT: READ CAREFULLY

The grade on the written exam is valid at most for three months.

Check the score of each exercise.

Please note that YES NO answers are NOT valid if you do not give an explanation.

SURNAME **NAME**

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SIGNATURE

GRADE

I solved the following exercise

1. ☐ **(Score 7)**
2. ☐ **(Score 5)**
3. ☐ **(Score 8)**
4. ☐ **(Score 6)**
5. ☐ **(Score 6)**

ORAL PART: questions

GRADE

Exercise 1. **(Score 8)** Consider the following nonlinear programming problem

$$\begin{array}{ll} \min_{x\in I\!R^3} & \frac{1}{2}x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_3 - x_2x_3 - 2x_1 - 3x_3 \\ & x_1 + x_2 + x_3 \leq 6 \\ & -2x_1 + x_2 = -1 \\ & -3x_1 - x_2 + x_3 \geq -3 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Unconstrained problem (score 2)

(i) **(score 1)** Consider the unconstrained problem (remove the constraints). Is there any unconstrained stationary point ? Which kind of point is it ?

(ii) **(score 1)** Consider the point $x^0 = (1,\ 1,\ 1)^T$ and write the first iteration of the gradient method with exact line search to obtain the new point x^1

Constrained problem (score 5)

(iii) **(score 0,5)** State if the problems is convex or concave or none of the two.

(iv) **(score 2)** Consider the point $\widehat{x} = (1,\ 1,\ 1)^T$ and write the KKT conditions in $\widehat{x}$. Evaluate the multipliers. Are the KKT conditions satisfied ?

(v) **(score 1)** Write the system to get a feasible and descent direction in $\widehat{x}$. Does a solution of the system (namely a feasible and descent direction) exist?

(vi) **(score 1,5)** Consider the point $x^0 = (1,\ 1,\ 1)^T$ and write the linear programming problem (in the variables x) to find the direction d^0 at the first iteration of the conditional gradient (Frank-Wolfe) method. Assume that the optimal solution of this Linear problem is $x^{0*} = (\frac{1}{2},\ 0,\ 0)^T$, write the direction d^0

Exercise 2. **(Score 4)** Consider the following Linear programming problem

$$\begin{array}{ll} \min & 2x_1 - 2x_2 + 2x_3 \\ & x_1 + x_2 + x_3 \leq 6 \\ & -2x_1 + x_2 = -1 \\ & -3x_1 - x_2 + x_3 \geq -3 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Consider the point $\widehat{x} = (1,\ 1,\ 4)^T$

(i) **(score 0,5)** Is the point $\widehat{x}$ a vertex ?

(ii) **(score 1,5)** Find a feasible direction along which it is possible to move from $\widehat{x}$ and find an additional active constraint besides those already active in $\widehat{x}$.

Check if the direction is a descent one.

Find the stepsize $t^{\max}$ and the corresponding new point y.

Is the point y a vertex ?

(iii) **(score 0,5)** Write the problem in the standard form for the simplex method.

(iv) **(score 1)** Write a BFS for the problems at point (iii) and specify the corresponding matrices B and N.

(v) **(score 0,5)** Write the auxiliary problem in (x, α) for the Phase I of the simplex method. Write any BFS for the auxiliary problem (only the values of the variables (x, α)).

Exercise 3. (Score 8) Consider the following Linear programming problem

$$\begin{aligned} \min \quad & 2x_1 - 2x_2 + 2x_3 \\ & -2x_1 + x_2 = -1 \\ & -3x_1 - x_2 + x_3 \geq -3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(NB It is not the same of the Exercise 2: one constraint is missing)

(i) (score 1.5) Write the dual problem

(ii) (score 2) Solve graphically the dual: plot the feasible region, the level lines of the objective function, identify graphically the solution and find its value.

(iii) (score 2) Using duality theory, state if the primal problem has an optimal solution and in the affirmative case find it.

(iv) (score 1) State how the value of the optimal solution change if the rhs of the first constraint change from -3 to $-3 + \varepsilon$ with $\varepsilon > 0$ and sufficiently small.

(v) (score 1) Consider the BFS corresponding to the optimal vertex x^* (namely find the values of the slack and/or surplus variables associated to the standard form of the given problem).
Find the values of the reduced costs γ.

Exercise 4. (Score 6) Consider the following integer linear programming problem

$$\begin{aligned} & \min \ z = 3x_1 + x_2 \\ & -3x_1 + 2x_2 \leq 2, \\ & x_1 + x_2 \geq 2 \\ & x_1 \leq 2, \\ & x_1, x_2 \geq 0, \quad x_1, x_2 \quad \text{integer} \end{aligned} \qquad (P)$$

Solve the problem using the Branch-and-Bound method and the graphical solution

Exercise 5. (Score 6)

A cargo plane has three compartments for storing cargo: front, centre and rear. These compartments have the following limits on both weight and space:

Compartment	Weight capacity (tonnes)	Space capacity (cubic metres)
Front	10	6800
Centre	16	8700
Rear	8	5300

The following four cargoes are available for shipment on the next flight:

Cargo	Weight (tonnes)	Volume (cubic metres/tonne)	Profit (£/tonne)
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

Any proportion of these cargoes can be accepted. Furthermore, the weight of the cargo in the centre compartments must be the same proportion with respect to front an rear compartments of the compartment's weight capacity to maintain the balance of the plane.

The objective is to determine how much of each cargo C1, C2, C3 and C4 should be accepted and how to distribute each among the compartments so that the total profit for the flight is maximised. Formulate the above problem as a linear program