OPERATIONS RESEARCH

EXAM February 3, 2020

IMPORTANT: READ CAREFULLY

The grade on the written exam is valid at most for three months.

Check the score of each exercise.

Please note that YES NO answers are NOT valid if you do not give an explanation.

SURNAME **NAME**

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SIGNATURE

GRADE

I solved the following exercise

1. ☐ **(Score 7)**
2. ☐ **(Score 5)**
3. ☐ **(Score 8)**
4. ☐ **(Score 6)**
5. ☐ **(Score 6)**

ORAL PART: questions

GRADE

Exercise 1. **(Score 8)** Consider the following nonlinear programming problem

$$\begin{aligned} \min_{x\in I\!R^3} \quad & x_1^2 + \frac{1}{2}x_2^2 + 2x_3^2 + 2x_2x_3 - x_1x_3 - 2x_2 - 3x_3 \\ & 2x_1 + 6x_2 + 6x_3 \geq 3 \\ & 18x_1 + 2x_2 - 10x_3 = 1 \\ & -4x_1 + 3x_2 + x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Unconstrained problem (score 2)

(i) **(score 1)** Consider the unconstrained problem (remove the constraints). Is there any unconstrained stationary point ? Which kind of point is it ?

(ii) **(score 1)** Consider the point $x^0 = (0,\ \frac{1}{2},\ 0)^T$ and write the first iteration of the gradient method with exact line search to obtain the new point x^1

Constrained problem (score 5)

(iii) **(score 0,5)** State if the problems is convex or concave or none of the two.

(iv) **(score 2)** Consider the point $\widehat{x} = (0,\ \frac{1}{2},\ 0)^T$ and write the KKT conditions in $\widehat{x}$. Evaluate the multipliers. Are the KKT conditions satisfied ?

(v) **(score 1)** Write the system to get a feasible and descent direction in $\widehat{x}$. Does a solution of the system (namely a feasible and descent direction) exist?

(vi) **(score 1,5)** Consider the point $x^0 = (0,\ \frac{1}{2},\ 0)^T$ and write the linear programming problem (in the variables x) to find the direction d^0 at the first iteration of the conditional gradient (Frank-Wolfe) method. Assume that the optimal solution of this Linear problem is $x^{0*} = (\frac{1}{2},\ 1,\ 1)^T$, write the direction d^0

Exercise 2. **(Score 4)** Consider the following Linear programming problem

$$\begin{aligned} \max \quad & 63x_1 + 33x_2 + 15x_3 \\ & 2x_1 + 6x_2 + 6x_3 \geq 3 \\ & 18x_1 + 2x_2 - 10x_3 = 1 \\ & -4x_1 + 3x_2 + x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Consider the point $\widehat{x} = (1,\ 0,\ \frac{17}{10})^T$

(i) (score 0,5) Is the point $\widehat{x}$ a vertex ?

(ii) (score 1,5) Find a feasible direction along which it is possible to move from $\widehat{x}$ and find an additional active constraint besides those already active in $\widehat{x}$.

Check if the direction is a descent one.

Find the stepsize $t^{\max}$ and the corresponding new point y.

Is the point y a vertex ?

(iii) (score 0,5) Write the problem in the standard form for the simplex method.

(iv) (score 1) Write a BFS for the problems at point (iii) and specify the corresponding matrices B and N.

(v) (score 0,5) Write the auxiliary problem in (x, α) for the Phase I of the simplex method. Write any BFS for the auxiliary problem (only the values of the variables (x, α)).

Exercise 3. (Score 8) Consider the following Linear programming problem

$$\begin{aligned} \max \quad & 63x_1 + 33x_2 + 15x_3 \\ & 2x_1 + 6x_2 + 6x_3 \geq 3 \\ & 18x_1 + 2x_2 - 10x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(NB It is not the same of the Exercise 2: one constraint is missing)

(i) (score 1.5) Write the dual problem

(ii) (score 2) Solve graphically the dual: plot the feasible region, the level lines of the objective function, identify graphically the solution and find its value.

(iii) (score 2) Using duality theory, state if the primal problem has an optimal solution and in the affirmative case find it.

(iv) (score 1) State how the value of the optimal solution change if the rhs of the first constraint change from 3 to $3 + \varepsilon$ with $\varepsilon > 0$ and sufficiently small.

(v) (score 1.5) State which is the maximum value of $\varepsilon > 0$ for which the analysis in point (iv) holds.

Exercise 4. (Score 6) Consider the following integer linear programming problem

$$\begin{aligned} \max \quad & z = 3x_1 + x_2 \\ & 2x_1 + 18x_2 \leq 63, \\ & 6x_1 + 2x_2 \leq 33 \\ & 6x_1 - 10x_2 \leq 15, \\ & x_1, x_2 \geq 0, \quad x_1, x_2 \quad \text{integer} \end{aligned} \qquad (P)$$

Solve the problem using the Branch-and-Bound method and the graphical solution for the LP relaxation

Exercise 5. (Score 6) The four sales departments (V1, V2, V3, V4) of a company require daily to be supplied by the three production departments (P1, P2, P3) with pre-defined quantities of a new product to be placed on the market. The table below shows the quantities (in quintals) required by each of the sales departments on a daily basis, together with unit costs (in euros per quintal) of the transport of one quintal of product from each of the productive departments to each one of sales departments.

	V1	V2	V3	V4
quantities required	250	380	420	195
Transportation cost	V1	V2	V3	V4
P1	9	7.5	8	8.5
P2	6.5	7	7.8	8
P3	7	6.7	8.2	7.9

The following table shows, for each production department, its maximum daily production capacity (expressed in quintals), the unit manufacturing cost (in euros per quintal).

	P1	P2	P3
maximum daily production	850	500	530
unit manufacturing cost	3	3.5	4.5

Furthermore each production department cannot serve more than three sales departments.

Write an ILP problem that satisfy the demand exactly and minimize the total cost of the firm.