OPERATIONS RESEARCH

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TEST PART.

Each correct answer gives a score of 0,5. You must answer AT LEAST 6 QUESTIONS CORRECTLY otherwise you get a penalty of -1

Consider the following optimization problem

$$\begin{array}{ll} \max_{x \in I\!R^4} & f(x_1, x_2, x_3, x_4) \\ & x_1 + x_2 - x_3 + 2x_4 \le 2 \\ & 3x_1 - 2x_2 + x_3 + x_4 = 3 \\ & x_1, x_2, x_3 \ge 0 \end{array}$$

Answer True (T) or False (F) to the following questions:

1. The feasible region of problem (P) is convex
2. Given any pair of feasible points $\widehat{x}$, $\tilde{x}$, the middle point $\frac{1}{2}\widehat{x} + \frac{1}{2}\tilde{x}$ can be NOT feasible
3. In the point $\tilde{x} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}$ we have two active constraints.
4. The point $\widehat{x} = \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \\ \frac{3}{2} \\ 0 \end{pmatrix}$ is a vertex.
5. The feasible region admits a vertex
6. When the function f is convex the Hessian matrix is positive semidefinite
7. When the function $f(x)$ is convex, Problem (P) is a concave optimization Problem
8. A feasible point $\hat{x}$ which satisfies $\nabla f(\hat{x}) = 0$ is a candidate to be a local minimizer
9. If the function f is linear, the global solution is on the boundary of the feasible region
10. When the feasible region is NOT bounded not empty, the problem is always unbounded

EXERCISE PART

Exercise 1. **(Score 7)** Consider the following problem

$$\begin{aligned} \min_{x\in \mathbb{R}^4} \quad & \frac{1}{2}x_1^2 + 2x_2^2 + x_3^2 + 3x_4^2 + 2x_1x_3 - x_2x_3 + 2x_2x_4 - 2x_1 - 3x_3 \\ & x_1 + x_2 - x_3 + 2x_4 \leq 2 \\ & 3x_1 - 2x_2 + x_3 + x_4 = 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(i) **(score 2)** State if the problems is convex or concave or none of the two.

(ii) **(score 2.5)** Consider the point $\tilde{x} = (1,\ 1,\ 2,\ 0)^T$ and write the KKT conditions in $\widehat{x}$.
Write the values of the multipliers.
Are the KKT conditions satisfied ?

(iii) **(score 2.5)** Given the point $\tilde{x} = (1,\ 1,\ 2,\ 0)^T$, find a feasible direction $\bar{d}$ along which it is possible to move from $\widehat{x}$ and find an additional active constraint besides those already active in $\widehat{x}$.
Check if the direction is a descent one.
Find the stepsize $t^{\max}$.
Write the new point $y = x + t^{\max}\bar{d}$.

Exercise 2. **(Score 8)** Consider the following Linear programming problem

$$\begin{aligned} \min \quad & 2x_1 + 3x_2 \\ & x_1 + 3x_2 \geq 3 \\ & -x_1 + 2x_2 \leq 5 \\ & x_1 - x_2 \leq 2 \\ & 2x_1 + x_2 \geq 2 \\ & x_1 \geq 0 \end{aligned} \qquad (P)$$

(i) **(score 2)** Solve graphically the problem: plot the feasible region, the level lines of the objective function, identify graphically the solution.
Report the optimal solution and the value of the objective function.

(ii) **(score 3)** Write the dual problem. Find, if it exists, a solution of the dual problem.
Report the optimal dual solution and the value of the objective function.

(iii) **(score 3)** Add the constraints

$$x_1, x_2 \quad \text{integer}$$

and solve the problem using the Branch-and-Bound method and the graphical solution for the LP relaxation.
Report the optimal integer solution and the value of the objective function.

Exercise 3. **(Score 6)** You must construct a diet with minimum cost using four food items. The diet chart that gives calories, protein, carbohydrate and fat content for 4 food items follows

	Food Item 1	Food Item 2	Food Item 3	Food Item 4
Calories	400	200	150	500
Protien (in grams)	3	2	0	0
Carbohydrates (in grams)	2	2	4	4
Fat (in grams)	2	4	1	5
Cost	€0.50	€0.20	€0.30	€0.80

The chart gives the nutrient content as well as the per-unit cost of each food item. The diet has to be planned in such a way that it should contain at least 500 calories, 6 grams of protein, 10 grams of carbohydrates and 8 grams of fat.

Write an LP problem that satisfy the requirements for the diet and minimize the total cost.