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**Optimal Incentives in a Principal-Agent  
Model with Endogenous Technology**

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# Optimal Incentives in a Principal-Agent Model with Endogenous Technology\*

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## Abstract

One of the standard predictions of the agency theory is that more incentives can be given to agents with lower risk aversion. In this paper we show that this relationship may be absent or reversed when the technology is endogenous and projects with a higher efficiency are also riskier. Using a modified version of the Holmstrom and Milgrom's (1987) framework, we obtain that lower agent's risk aversion unambiguously leads to higher incentives when the technology function linking efficiency and riskiness is elastic, while the risk aversion-incentive relationship can be positive when this function is rigid.

**Keywords:** principal-agent; incentives; risk aversion; endogenous technology.

**JEL Classification:** D82.

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# 1 Introduction

One of the main results of the agency theory is the trade-off between incentives and insurance. Lower agent's risk aversion allows the principal to provide more incentives by making the payment of the agent more related to output while higher uncertainty increases the gains from insuring the agent and reduces the pay-for-performance sensitivity. The empirical works testing the link between uncertainty and incentives have found mixing results however (e.g., Rao and Hanumantha, 1971; Allen and Lueck, 1995; Aggarwal and Samwick, 1999; Core and Guay, 2002; Wulf, 2007). In many cases, the empirical findings are even in contradiction with the standard predictions of the theory as they document a positive (rather than negative) correlation between observed measures of uncertainty and the provision of incentives (see Prendergast, 2002, for an extensive discussion on this point).

Recently the matching literature (e.g., Wright, 2004; Legros and Newman, 2007; Serfes, 2005, 2008; Li and Ueda, 2009) has attempted to provide a justification of the above results based on the endogenous matching between principals and agents. For instance, Serfes (2005) shows that, whereas under efficient *positive assortative matching* (in which higher risk-averse agents are optimally matched with riskier principals) the traditional trade-off between risk and incentives holds, under efficient *negative assortative matching* (lower risk-averse agents are matched with riskier principals) this trade-off can fail to hold, in particular when matching curves are very steep. Li and Ueda's (2009) show, instead, that if the agents differ only in their productivity, safer firms will offer high-powered incentives schemes, in this way capturing the higher productive workers at the endogenous matching.

While useful in disentangling the direct and indirect effects of risk on equilibrium contracts, the endogenous matching models adopt a number of highly simplifying assumptions, as the monotonicity of equilibrium matching patterns. For this reason, in this paper we propose an alternative and simpler explanation of the relationship between risk and incentives, one based on the endogeneity of the technology. In particular, we show that the traditional relationship between agent's risk aversion and optimal incentive may be absent or reversed when the technology is endogenous and projects with a higher efficiency are also riskier. More specifically, using a modified version of the Holmstrom and Milgrom's (1987) framework, we obtain that lower agent's risk aversion unambiguously leads to higher incentives only when the technology function linking risk and efficiency is elastic, while the risk aversion-incentive relationship can be positive when this function is rigid. This is because a lower risk aversion of the agent makes it optimal for the principal the adoption of a riskier and a more efficient technology. While the higher efficiency of the new technology (as well as the lower agent's risk aversion) allows the principal to give more incentives to the agent, its higher riskiness makes the provision of incentives more costly which works in the direction of reducing the optimal degree of the pay-for-performance sensitivity. We also show that differently from the endogenous matching models (e.g. Serfes, 2005) the positive risk-incentive relationship is compatible with both positive and negative assortative matchings between principals

and agents.

The paper is organized as follows. In Section 2 we describe the framework and Section 3 provides the solution of the model. Section 4 presents the comparative statics analysis of the effect of a reduction of the agent's risk aversion on incentives. Section 5 concludes.

## 2 The Framework

We consider a moral hazard model as in Holmstrom and Milgrom (1987). The principal owns the technology and is risk neutral. The agent is risk averse and has a constant absolute risk aversion (CARA) utility function with a coefficient of absolute risk aversion equal to  $r$ . Total output is equal to

$$y = e + \varepsilon, \quad (1)$$

where  $e$  is the agent's action (e.g., effort) and  $\varepsilon$  is an (unobservable) random variable normally distributed with zero mean and variance  $\sigma^2$ . The technology is characterized by quadratic costs, so that the agent's cost of action is

$$c(e) = \frac{k}{2}e^2, \quad (2)$$

where  $k$  is a constant representing the efficiency of the technology employed. Better technologies are characterized by a lower  $k$  and vice-versa. The agent's reservation utility is equal to  $\delta$ .

We here modify the Holmstrom and Milgrom's framework by assuming the existence of a given set of technologies (or projects) with different levels of efficiency and riskiness among which the principal can choose. In particular, we assume a trade-off between efficiency and riskiness so that technologies with a higher volatility  $\sigma^2$  also have a lower marginal cost of effort, i.e.,

$$k \equiv k(\sigma^2) \quad \text{with} \quad k' \equiv \frac{dk}{d\sigma^2} < 0, \quad (3)$$

where  $k > 0$  for all  $\sigma^2 \in (0, \infty)$ . For simplicity,  $k(\cdot)$  is assumed to be a function continuous and differentiable in  $\sigma^2$ .

In this framework, the principal decides the optimal technology and the agent's payment scheme; then, the agent optimally chooses the action. In the next sections, we determine these choices and analyze the effects of a variation of the agents' risk aversion on the optimal payment scheme of the agent.

### 3 The Equilibrium

We solve the problem by determining the optimal payment scheme and the agent's action for a given technology.<sup>1</sup> Then, we determine the optimal technology choice of the principal.

Holmstrom and Milgrom (1987) show that a linear payment is optimal in the above framework, so that the agent's payoff can be written as  $s(y) = \beta y + \alpha$ , where  $\alpha$  and  $\beta$  are constants optimally chosen by the principal that have to be determined. Taking into account (1), (2) and the distribution of the shock, the agent's expected utility is

$$E \{ -\exp \{ -r [s(y) - c(e)] \} \} = -\exp \{ -r [\beta e + \alpha - (1/2)ke^2 - (1/2)r\beta^2\sigma^2] \},$$

and therefore his maximization problem can be written as

$$\max_e \beta e + \alpha - (1/2)ke^2 - (1/2)r\beta^2\sigma^2. \quad (4)$$

The first order condition of this problem is  $\beta = ek$ . Substituting this condition into (4) and then setting the expression (the agent's certainty equivalent) equal to  $\delta$  gives  $\alpha = -(1/2)ke^2 + (1/2)r\beta^2\sigma^2 + \delta$ . Hence, the principal's maximization problem becomes

$$\max_e \pi = E[y - s(y)] = e - (1/2)ke^2 - (1/2)rk^2e^2\sigma^2 - \delta, \quad (5)$$

which gives the following well-known second best solution for the agent's action<sup>2</sup>

$$e^* = \frac{1}{k(1 + rk\sigma^2)}. \quad (6)$$

Using the fact that  $\beta = ek$ , it follows that the optimal share of output paid to the agent is

$$\beta^* = \frac{1}{1 + rk\sigma^2}, \quad (7)$$

and the optimal fix payment is

$$\alpha^* = \frac{-1 + rk\sigma^2}{2k(1 + rk\sigma^2)^2} + \delta. \quad (8)$$

Let now  $\sigma_*^2$  denote the variance of the optimal project. This is the solution of the following maximization problem of the principal

$$\max_{\sigma^2} \pi^* = \frac{1}{2k(1 + rk\sigma^2)} - \delta, \quad (9)$$

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<sup>1</sup>We here omit some details of the analysis as the complete description of the solution can be found in Holmstrom and Milgrom (1987).

<sup>2</sup>The first order condition of problem (5) is  $d\pi/de = 1 - ke - rk^2e\sigma^2 = 0$  and the second order condition is always satisfied as  $d^2\pi/de^2 = -k - rk^2\sigma^2 < 0$ .

subject to the technological constraint (3).<sup>3</sup> The first order condition of this problem is

$$\frac{d\pi^*}{d\sigma^2} = -\frac{k' + 2rkk'\sigma^2 + rk^2}{2k^2(1 + rk\sigma^2)^2} = 0, \quad (10)$$

and therefore the variance  $\sigma_*^2$  of the optimal project is implicitly defined by the following equation

$$F \equiv -k' - 2rkk'\sigma^2 - rk^2 = 0, \quad (11)$$

where  $k \equiv k(\sigma^2)$  and  $k' \equiv k'(\sigma^2)$ . The effort cost parameter at the optimal technology follows from (3) and it is  $k(\sigma_*^2)$ .<sup>4</sup>

In order to have unique maximum, which will be useful for the comparative static analysis, we restrict the attention to functions of the technology  $k(\sigma^2)$  such that  $F$  in (11) is strictly concave. This requires that the following condition is always satisfied

$$dF/d\sigma^2 = -4rkk' - 2r(k')^2\sigma^2 - k''(1 + 2rk\sigma^2) < 0. \quad (12)$$

The first component of (12) is positive (as  $k' < 0$ ), the second is negative while the third one has the opposite sign of  $k''$ . Therefore, while  $k(\sigma^2)$  can generally be concave or convex, a *sufficient* condition for (12) to hold is that  $k$  is sufficiently convex, i.e., that  $k''$  is positive and large enough. The following proposition summarizes these results.

**Proposition 1** *The principal chooses the technology with the variance  $\sigma_*^2$  implicitly defined by equation (11) and efficiency  $k(\sigma_*^2)$  as in (3). The agent optimally chooses the action  $e^*$  reported in (6) and the coefficients of the linear payment scheme  $\beta^*$  and  $\alpha^*$  are defined respectively by (7) and (8) with  $k \equiv k(\sigma_*^2)$  and  $\sigma^2 \equiv \sigma_*^2$ .*

## 4 Agent's risk aversion and the provision of incentives

We now analyze how a variation in the agent's risk aversion affects the provision of incentives when, as in our framework, such a variation also induces a change in the technology adopted.

By applying the implicit function theorem to equation (11), we obtain that

$$\frac{\partial \sigma_*^2}{\partial r} = -\frac{\partial F / \partial r}{\partial F / \partial \sigma^2} = -\frac{-2rkk'\sigma^2 - k^2}{-4rkk' - k'' - 2rk'k'\sigma^2 - 2rk k''\sigma^2} < 0, \quad (13)$$

as the denominator is negative from the second order condition of maximization problem (9) and the numerator is also negative since the first order condition (11) implies that  $-2rkk'\sigma^2 - k^2 = k'/r < 0$ . This means that a reduction in the agent's risk aversion

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<sup>3</sup>The maximized expected profit  $\pi^*$  (for a given technology) is obtained from the substitution of (6) into (5).

<sup>4</sup>Note that the first two components of (11) are positive while the third one is negative.

increases the riskiness  $\sigma_*^2$  as well as the efficiency ( $k(\sigma_*^2)$  goes down) of the technology chosen by the principal.

We will now show that while the reduction of the agent's risk aversion induces the principal to provide more incentives by increasing the agent's payment related to the output for any *given* technology (it is immediate from (7) that  $\beta^*$  is decreasing in  $r$ ), this may no longer hold if the lower risk aversion of the agent leads the principal to change the technology employed (i.e., its efficiency and riskiness). In this case the characteristics of the new technology may affects the optimal provision of incentives in ways that counterbalance the former effect.

The total effect of a reduction of the agent's risk aversion on the optimal share  $\beta^*$  of output paid to the agent is obtained by total differentiation of (7) which gives

$$\frac{d\beta^*}{dr} = \underbrace{\frac{\partial\beta^*}{\partial r}}_{\text{direct effect } (-)} + \underbrace{\frac{\partial\beta^*}{\partial\sigma^2} \frac{\partial\sigma^2}{\partial r}}_{\text{indirect riskiness effect } (+)} + \underbrace{\frac{\partial\beta^*}{\partial k} \frac{\partial k}{\partial\sigma^2} \frac{\partial\sigma^2}{\partial r}}_{\text{indirect efficiency effect } (-)}. \quad (14)$$

The first component in (14) represents the *direct effect* of a reduction of  $r$  on  $\beta^*$ , namely the effect on  $\beta^*$  if the same technology is employed. This component is equal to

$$\frac{\partial\beta^*}{\partial r} = -\frac{k\sigma^2}{(1 + rk\sigma^2)^2}, \quad (15)$$

and it is always negative as a lower risk aversion makes it optimal for the principal to give more incentives and less insurance to the agent, which requires increasing the payment related to output.

The other two components in (14) represent the *indirect effect* of the reduction of  $r$  on  $\beta^*$ , i.e. the effect caused by a change in the technology employed by the principal. The new technology is characterized by a higher efficiency and a higher riskiness which generate two opposing effects on  $\beta^*$ . The higher riskiness  $\sigma_*^2$  of the project makes it optimal the provision of more insurance and less incentives to the agent, and this implies that the payment related to output decreases (we can call this the *riskiness effect*). Indeed, we obtain that

$$\frac{\partial\beta^*}{\partial\sigma^2} = -\frac{rk}{(1 + rk\sigma^2)^2} < 0. \quad (16)$$

On the other hand, the new technology is also characterized by a higher efficiency (i.e., a lower cost of effort  $k$ ), which makes it optimal an increase of incentives as<sup>5</sup>

$$\frac{\partial\beta^*}{\partial k} = -\frac{r\sigma^2}{(1 + rk\sigma^2)^2} < 0. \quad (17)$$

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<sup>5</sup>This effect goes in the same direction of the direct effect generated by the reduction of  $r$ .

This means that  $\beta^*$  increases as  $r$  goes down. We call this the *efficiency effect* and it goes in the same sign of the direct effect.<sup>6</sup>

Therefore, the net indirect effect due to the change of technology may in general lead to an increase or a decrease in  $\beta^*$ . We now try to understand under what conditions there is a definite sign in the relationship between  $r$  and  $\beta^*$ .

Let us first analyze the case where the net indirect effect has the same sign of the direct effect, so that  $d\beta^*/dr$  is always negative and, therefore, a lower agent's risk aversion leads to more incentives. From (14) it is immediate that this is the case when  $(\partial\beta^*/\partial\sigma^2) + (\partial\beta^*/\partial k)k' \geq 0$  since  $\partial\sigma^2/\partial r$  is always negative. Using (16) and (17), we obtain that this condition is satisfied when the elasticity  $E_{k\sigma}$  of the technology with respect to the volatility is weakly greater than 1, i.e.,

### Condition 1

$$E_{k\sigma} \equiv -k' \frac{\sigma^2}{k} \geq 1.$$

The intuition for this result is the following. If the function  $k(\sigma^2)$  is elastic, then the increased efficiency of the technology (i.e., the reduction of  $k$ ) associated to a given increase in its riskiness  $\sigma^2$  is relatively large. This implies that the efficiency effect dominates the riskiness effect. Therefore, under Condition 1, the indirect effect has a negative sign and the reduction of the agent's risk aversion  $r$  always leads to an increase of  $\beta^*$ , which means that the principal will provide more incentives to the agent.

When  $k(\sigma^2)$  is rigid and therefore Condition 1 does not hold, the efficiency effect is small relative to the riskiness effect and the indirect effect will be positive. As the direct effect has a negative sign, the total effect of a reduction in  $r$  on  $\beta^*$  will generally be ambiguous. However, if the increased riskiness of the new technology is sufficiently strong, then the lower agent's lower may induce a reduction of incentives. The following proposition summarizes these results.

**Proposition 2** *A reduction in the agent's risk aversion  $r$  generates two effects on the optimal share of output  $\beta^*$  paid to the agent. The direct effect always increases  $\beta^*$  while the indirect effect due to the change of technology can lead to an increase or a decrease of  $\beta^*$ . When Condition 1 is satisfied, both the direct and indirect effects have the same sign and a lower risk aversion  $r$  unambiguously increase  $\beta^*$  (i.e.,  $\partial\beta^*/\partial r < 0$ ). When Condition 1 does not hold, the total effect of  $r$  on  $\beta^*$  is generally ambiguous.*

It can be interesting to compare the above results with those usually obtained in the endogenous matching models. However, since the two models strongly differ in their

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<sup>6</sup>A strsigthforward comparison shows that, differently from our model, the endogenous matching models only consider the direct and the indirect *riskiness effect* (e.g. Serfes, 2005), but not the indirect *efficiency effect*. Thus, whereas under *positive* assortative matchings the riskiness effect is negative (since riskier principals attract more risk-averse agents) under *negative* assortative matchings the indirect riskiness effect is positive (since now riskier principals are matched with less risk-averse agents) and the final effect of risk on incentives may, in this case, be ambiguous.



assumptions, they are hardly comparable. What can be done here is to consider the basic version of the matching model introduced by Serfes (2005) and add to it, as in our model, an inverse link between technology and risk. As in Serfes (2005), we can assume that the principals are uniformly distributed according to their riskiness in the interval  $[\sigma_L^2, \sigma_H^2]$  and the agents in  $[r_L, r_H]$  according to their risk aversion.<sup>7</sup> Now, by taking into account the effect of function  $k(\cdot)$ , the condition ensuring a *positive* (*negative*) assortative matching becomes

$$\frac{\partial \pi}{\partial \sigma^2 \partial r} \geq (\leq) 0 \Leftrightarrow \frac{kr\sigma^2 + 2r(\sigma^2)^2 k' - 1}{2(kr\sigma^2 + 1)^3} \geq (\leq) 0 \text{ holding for } r\sigma^2 \geq (\leq) \underbrace{\frac{1}{k + 2\sigma^2 k'}}_{(-)} \quad (18)$$

instead of (as in Serfes 2005, p. 346):

$$\frac{\partial \pi}{\partial \sigma^2 \partial r} \geq (\leq) 0 \Leftrightarrow \frac{kr\sigma^2 - 1}{2(kr\sigma^2 + 1)^3} \geq (\leq) 0 \text{ holding for } r\sigma^2 \geq (\leq) \frac{1}{k}. \quad (19)$$

Comparing (18) and (19) helps to make clear that, in a matching model modified to include an inverse relationship between technology and risk, the condition required for a positive (negative) assortative matching becomes more (less) demanding than when technology is given. As a result, it becomes now harder, in this model, to obtain a well-behaved (negative) relationship between risk and incentives.<sup>8</sup>

However, comparing our model without endogenous matching with the modified version of Serfes's (2005) model, it can be seen that Condition 1 is in general compatible with both positive and negative assortative matchings. This is not surprising since, differently from the usual matching models in which only the direct and riskiness effect are at work, our model introduces an additional effect (denoted *efficiency effect*) that works in the same direction as the direct effect (see expression 4.2 and footnote 5 on this point).

To see this point, let us consider a specific functional form for the relationship between the cost parameter  $k$  of the agent and the risk of the project expressed by  $\sigma^2$ . In particular, let us assume that this technology function has a constant elasticity and it is given by  $k = (\sigma^2)^{-\eta}$ , with  $\eta \in (0, 1/2)$  and  $\sigma^2 \in (0, \infty)$  so that  $k$  is finite and positive for all  $\sigma^2$ . Then,  $k' = -\eta k(\sigma^2)^{-1} < 0$  and  $k'' = \eta(\eta + 1)k(\sigma^2)^{-2} > 0$ .

The first order condition (11) of the principal's maximization problem can be rewritten as

$$\eta (\sigma^2)^{\eta-1} + r(2\eta - 1) = 0, \quad (20)$$

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<sup>7</sup>See also Legros and Newman (2007).

<sup>8</sup>In a matching model (with fixed or variable technology) it remains true that a well-behaved relationship between risk and incentives is guaranteed only under *positive* assortative matching (see also footnote 5).

which implies that the variance of the optimal technology is equal to<sup>9</sup>

$$\sigma_*^2 = \left[ \frac{\eta}{r(1-2\eta)} \right]^{\frac{1}{1-\eta}}. \quad (21)$$

From  $\eta < 1/2$  follows that Condition 1 is not satisfied (as  $E_{k\sigma} = \eta < 1$ ) and the indirect effect is positive, i.e., the change of technology induced by the lower agent's risk aversion  $r$  leads to a reduction of  $\beta^*$  (the riskiness effect dominates the efficiency effect). This indirect effect opposes to the direct effect which instead pushes for an increase in  $\beta^*$ . The total effect of a reduction of  $r$  on  $\beta^*$  can be computed by substituting (15), (16), (17) and  $\partial\sigma_*^2/\partial r$  (which is obtained from (21)) into (14). This leads to  $\partial\beta^*/\partial r = 0$  which means that, in this special case, the direct and indirect effect of a change in  $r$  on  $\beta^*$  exactly offset each other and therefore that a reduction in the agent's risk aversion leaves the fraction of output paid to the agent unchanged. Moreover, from (20) it is obtained that

$$\frac{\partial\pi}{\partial\sigma^2\partial r} = \eta(\sigma^2)^{\eta-1} + (2\eta - 1) \quad (22)$$

which can be either positive or negative for  $\eta < 1/2$  (e.g., if  $\sigma^2 = 1$ , the expression is positive for  $\eta > 1/3$  and negative for  $\eta < 1/3$ ) and, therefore, is in general compatible with both positive and negative assortative matchings, as discussed above.

## 5 Conclusions

We have shown that in a principal-agent model with endogenous technology choice the usual negative trade-off existing between the agent's risk aversion and the optimal incentive does not necessarily hold and can, in some cases, be reversed. This may occur when the link between the efficiency of the technology and its riskiness is weak. The reason is that for higher levels of agent's risk aversion the principal can decide to select less risky (and not much more inefficient) technologies that, in turn, make convenient the adoption of more powerful incentive schemes.

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<sup>9</sup>First note that  $\eta < 1/2$  is necessary in order to get an interior solution. Second, it is immediate that  $d\pi^*/d\sigma^2 \gtrless 0$  if  $\sigma^2 \gtrless \sigma_*^2$ . This means that profits are monotonically increasing in  $\sigma^2$  when  $\sigma^2 < \sigma_*^2$  and monotonically decreasing when  $\sigma^2 > \sigma_*^2$ , which confirms that profits are maximum at  $\sigma_*^2$ .

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