Nonlinear Optimization: a bridge from theory to applications
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ABSTRACTS of the CONTRIBUTED LECTURES
Unsupervised Visual Object Categorization by Sparse PCA

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The unsupervised visual object categorization problem attempts to uncover the category information of a given image database without relying on any information on the image content. We develop a technique based on Sparse PCA, which uses the first few sparse principal components to pick up object categories. Our experiments show that this technique has great potential in selecting the categories correctly. We solve the Sparse PCA problem with an alternative maximization algorithm, and show that this outperforms the existing algorithms.

Joint work with N-M. Cheung, P. Richtárik, and Martin Takáč

An Optimal Control Framework for Piecewise Deterministic Processes

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A new framework for the optimal control of piecewise deterministic processes (PDP) is presented. PDPs are a class of stochastic processes where the randomness affects a deterministic motion only at time epochs, that are generated by a discrete Markov process. PDPs can be described statistically by time dependent marginal probability density functions (PDF) that are governed by first-order hyperbolic partial differential equations (PDE). In this novel optimal control framework the PDFs are used to build the objective function and the minimization problem corresponds to solving an optimality system of hyperbolic PDEs. A conservative and positive preserving numerical method is used to approximate the optimality system. The effectiveness of this methodology is tested on application problems in the scenario of non linear model predictive control strategies formulated as a sequence of open-loop optimality systems.

This is a joint work with Alfio Borzi (U. Wuerzburg) and it is supported in part by the EU Marie Curie International Training Network Multi-ITN STRIKE Projekt Novel Methods in Computational Finance and in part by the ESF OPTPDE Programme. The author also acknowledges financial support from Indam-GNCS.
Forward–Partial Inverse Method for Solving Monotone Inclusions: Application to Land-Use Planning

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In this talk we provide a method for solving the monotone inclusion

\[ \text{find } x \in H \text{ such that } 0 \in N_V x + Ax + Bx, \]

where \( H \) is a real Hilbert space, \( V \) is a closed vectorial subspace of \( H \), \( N_V \) is the normal cone to \( V \), \( A: H \to 2^H \) is a maximally monotone operator, and \( B: H \to H \) is a cocoercive operator on \( V \). The proposed method exploits the structure of the problem by activating explicitly the operator \( B \) and by taking advantage of the vectorial subspace involved. The algorithm is a particular case of the forward-backward splitting when the proximal step includes the partial inverse of \( A \) with respect to \( V \). In the particular case when \( B \equiv 0 \), the algorithm becomes the method of partial inverses (Spingarn,1983), which solves

\[ \text{find } x \in H \text{ such that } 0 \in N_V x + Ax. \]

On the other hand, when \( V = H \), the algorithm reduces to the forward-backward splitting (Combettes,2004), which solves

\[ \text{find } x \in H \text{ such that } 0 \in Ax + Bx. \]

In addition, we deduce relation with a forward-Douglas-Rachford method and we derive the method proposed by Raguet, Fadili, and Peyré as a particular case. Finally, in a variational framework we apply the method for computing the optimal subsidies in order to obtain a desired land-use allocation.

Systems of Structured Monotone Inclusions: Duality, Algorithms, and Applications

Patrick Combettes

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A general primal-dual splitting algorithm for solving systems of structured coupled monotone inclusions in Hilbert spaces is introduced and its asymptotic behavior is analyzed. Each inclusion in the primal system features compositions with linear operators, parallel sums, and Lipschitzian operators. All the operators involved in this structured model are used separately in the proposed algorithm, most steps of which can be executed in parallel. This provides a flexible solution method applicable to a variety of problems beyond the reach of the state-of-the-art. Several applications are discussed to illustrate this point, in particular in the areas of image recovery and machine learning.
Using SVM for Combining Continuous Heuristics for Global Optimization Problems

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We consider a Support Vector Machine (SVM) based approach to combine different heuristics to solve the standard quadratic optimization problem (StQP). In literature different unconstrained formulations have been proposed in connection with simple multistart global scheme. None of them dominates the others in terms of best value found. We propose to combine the three heuristics using SVM to select both the heuristic and the starting point to be used in the multistart framework. To test our method we use StQP deriving from the Maximum Clique problems in the DIMACS challenge collection.

Joint work with L. Palagi

Yield Optimization in Electronic Circuits Design

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In electronic circuits production the yield is the percentage of circuits that satisfies all performance features, in different operating conditions and despite of random process variations. The yield optimization turns out to be the black box optimization of a function evaluated by expensive circuit simulations whose derivatives are not available. In the context of the MODERN European Research Program aiming at improving the production processes of the electronic industry, we propose an approach to yield optimization that combines the use of a Support Vector Machine as surrogate model of the circuit and of a Mixed-Integer Derivative Free optimization algorithm as black box optimization tool. As an example, we report the results obtained in the design of a DC-DC converter, compared with the results obtained using the commercial software currently adopted by ST-Microelectronic for this kind of applications.

Joint work with A. Ciccazzo, V. Latorre, G. Liuzzi, S. Lucidi and F. Rinaldi
A New Error Bound Result for Generalized Nash Equilibrium Problems and its Algorithmic Application

AXEL DREVES

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A new algorithm for the solution of Generalized Nash Equilibrium Problems is presented. This hybrid method combines the robustness of a potential reduction algorithm and the local quadratic convergence rate of the LP-Newton method. The local convergence theory is based on an error bound, and a new sufficient condition for it to hold is provided. This condition is weaker than known ones, and neither implies local uniqueness of a solution nor strict complementarity.


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We are interested in optimal control problems with pointwise state constraints governed by quasilinear partial differential equations. The nonlinearities include Nemytskii operators depending on the state variable and their weak deviation. The solution theory of the PDE itself is based on well known theory for pseudomonotone operators and comparison principles. With similar techniques we show the existence of a solution of the optimal control problem and convergence results of a direct method in a semi-discretized version.

Conditions Providing the Emptiness of the Cones of Generalized Support Vectors

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In [1], a linear separability criterion for sets of Euclidean space have been formulated. In the paper, there were defined the different cones of generalized support vectors (briefly, GSV–s) and studied their principal properties. In particular, it was shown that there exists a close relationship between the introduced cones of GSV-s and some widely known in the literature cones such as the polar cone, and so on.
The cones of GSV-s have applications in a wide range of nonlinear optimization problems. This stipulate the importance of studying the theoretical and computational aspects of checking up the cones of GSV-s on nonsingularity. Therefore, we are interested in conditions which ensure that the above-mentioned cones are empty, or, vice versa, they are nonempty [2]. We underline that the linear separability criterion allows to properly verify the cones of GSV-s on emptiness or non-emptiness [3].

We formulate and prove the necessary and sufficient conditions under which the cones of GSV-s for an arbitrary set of Euclidean space are empty.

References


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**Solving a Bi-Objective Facility Location Problem via a New Multi-Objective Evolutionary Algorithm**

**ARÁNZAZU GILA ARRONDONO**

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Recently, a new multi-objective evolutionary algorithm, called FEMOEA, has been proposed. This new algorithm can be applied to many nonlinear multi-objective optimization problems adapting a part of the algorithm, the improving method, which allows to improve the efficiency of points. FEMOEA also includes a new stopping rule, which allows to stop the algorithm as soon as a good approximation of the Pareto-front is available. FEMOEA has been compared to the reference NSGA-II and SPEA2 algorithms (see [1]). Comprehensive computational studies have shown that, among the studied algorithms, FEMOEA provides better approximations of the complete Pareto-front.

In this work, the bi-objective facility location and design problem presented in [2] is considered. A franchise wants to increase its presence in a given geographical region by opening one new facility. Both the franchisor (the owner of the franchise) and the franchisee (the actual owner of the new facility to be opened) have the same objective: maximizing their own profit. However, the maximization of the profit obtained by the franchisor is in conflict with the maximization of the profit obtained by the franchisee. The knowledge of the complete efficient set and the corresponding Pareto-front may be of help for both the franchisor and the franchisee in their negotiations to decide the final location for the new facility.

FEMOEA has been applied for solving this problem. In order to do that, a local search, which uses gradient information to improve the quality (efficiency) of the points, has been included in FEMOEA. The robustness of FEMOEA has been checked by analyzing its behaviour in small instances, for which an interval B&B method is able to provide the enclosure of the true Pareto-front (and the efficient set). The computational studies show that the heuristic method is competitive, being able to reduce, in average, the computing time of the exact method by approximately 99%, and offering high quality final approximations of the Pareto-front (and the efficient set).
Joint work with J. Fernández, J.L. Redondo and P.M. Ortigosa

References


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Primal Convergence From Dual Subgradient Methods for Convex Optimization

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When solving a convex optimization problem through a Lagrangian dual reformulation subgradient optimization methods are favourably utilized, since they often find near-optimal dual solutions quickly. However, an optimal primal solution is generally not obtained directly through such a subgradient approach. We construct a sequence of convex combinations of primal subproblem solutions, a so-called ergodic sequence, which is shown to converge to an optimal primal solution when the convexity weights are appropriately chosen. We generalize previous convergence results from linear to convex optimization and present a new set of rules for constructing the convexity weights defining the ergodic sequence of primal solutions. In contrast to rules previously proposed, they exploit more information from later subproblem solutions than from earlier ones. We evaluate the proposed rules on a set of nonlinear multicommodity flow problems and demonstrate that they clearly outperform the previously proposed ones.

Joint work with M. Patriksson and A. B. Strömberg
From Eckart & Young Approximations
to Moreau Envelopes and Vice Versa

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In matricial analysis and statistics, the so-called theorem of Eckart & Young (or of Eckart-Young-Mirsky-Schmidt) provides a best approximation of an arbitrary matrix by a matrix of rank at most $r$. It is based upon the singular value decompositions of a matrix. On the other hand, in variational analysis or optimization, the Moreau envelopes (or Moreau-Yosida approximations-regularizations) of a function provide a sequence of continuous approximations of it. This way of doing is very much in vogue in optimization, for theoretical purposes as well as for algorithmic ones (the so-called proximal methods). We prove here that we can go forwards and backwards between the two procedures, thereby showing that they carry essentially the same information.

The underlying motivation is the treatment of rank minimization problems, currently a hot topic in optimization.

References


The Effect of Hessian Evaluations
in the Global Optimization αBB Method

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To find a tight convex underestimator of an objective function is an essential problem in global optimization since it enables to easily compute a lower bound on the global optimal value, among others. In particular, it plays a crucial role in the well-known global optimization αBB method and its variants [1,2], which have been successfully applied in solving many real-life problems in system identification, optimal control, reactor network synthesis, and biology and chemistry (peptide and protein folding, parameter estimation of dynamic biological systems, chemical equilibrium problems, or molecular structure and cluster prediction).

The αBB method is based by augmenting the original nonconvex function by a relaxation term that is derived from an interval enclosure of the Hessian matrix. In our contribution, we discuss the advantages of symbolic computation of the Hessian. Symbolic computation often allows simplifications of the resulting expressions, which in turn implies less conservative underestimators. We show by numerical examples that even a small manipulation with the symbolic expressions, which can be processed automatically by computers, can have a large effect on the quality of underestimators.
A Superlinearly Convergent SQP Algorithm to Second-Order Optimal Points

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The primal-dual augmented Lagrangian merit function provides a link between augmented Lagrangian and stabilized SQP methods. In this talk I discuss an algorithm that incorporates this merit function in a procedure that includes a direction of descent and a direction of negative curvature. The use of the direction of negative curvature allows global convergence to points satisfying the second-order necessary optimality conditions to be shown. Furthermore, it can be proven that asymptotically, the algorithm becomes equivalent to stabilized SQP and is, therefore, superlinearly convergent to a primal-dual solution under weak regularity assumptions.

Joint work with P. Gill and D. Robinson

VI-Constrained Hemivariational Inequalities: Distributed Algorithms and Power Control in ad-hoc Networks

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We consider centralized and distributed algorithms for the numerical solution of a hemivariational inequality (HVI) where the feasible set is given by the intersection of a closed convex set with the solution set of a lower-level monotone variational inequality (VI). The algorithms consist of a main loop wherein a sequence of one-level, strongly monotone HVIs are solved that involve the penalization of the non-VI constraint and a combination of proximal and Tikhonov regularization to handle the lower-level VI constraints. The methods developed are used to successfully solve a new power control problem in ad-hoc networks.

Joint work with F. Facchinei, J.S. Pang, G. Scutari

References


Canonical Duality Theory Approach for the Data Clustering Problem

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In this work a new primal-dual strategy is proposed for solving the classical k-means clustering problem. This strategy is based on the canonical duality theory, a very useful tool in non convex optimization, that reformulates the original bi-level mix integer problem as a continuous min-max problem without duality gap. By using this reformulation and the triality theory developed recently, global optimal solution can be obtained for the original integer global minimization problem under certain conditions.

Joint work with D. Y. Gao

Exploiting Derivative-Free Local Searches in DIRECT-type Algorithms for Global Optimization

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In this paper we consider bound constrained global optimization problems where first-order derivatives of the objective function can be neither computed nor approximated explicitly. For the solution of such problems the DIRECT Algorithm has been proposed which has strong convergence properties and a good ability to locate promising regions of the feasible domain. However, the efficiency of DIRECT deteriorates as the dimension and the ill-conditioning of the objective function increase. To overcome these limits, we propose DIRECT-type algorithms enriched by the efficient use of derivative-free local searches combined with nonlinear transformations of the feasible domain and, possibly, of the objective function. We report extensive numerical results both on test problems from the literature and on an application in structural proteomics.

Joint work with G. Liuzzi and V. Piccialli
Bounds and Approximations in Multistage Stochastic Programming

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Multistage stochastic programs, which involve sequences of decisions over time, are usually hard to solve in realistically sized problems. Providing bounds for their optimal solution, may help in evaluating whether it is worth the additional computations for the stochastic program versus simplified approaches. In this talk we generalize the value of information gained from deterministic, pair solutions and rolling-horizon approximation in the two-stage linear case [1,2] to the multistage stochastic formulation both in the linear [3] and non-linear cases [5]. We show that theorems proved for two stage case are valid also in the multi-stage case. New measures of quality of the average solution are of practical relevance [4]. Numerical results on a transportation problem illustrate the relationships.

References


Joint work with E. Allevi and M. Bertocchi

A new Algorithm for the Solution of Large-Scale Singly Linearly Constrained Problems Subject to Simple Bounds

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Many engineering and economic applications can be formulated by a minimization problem subject to a single linear constraint and bounds on the variables and the majority of them are characterized by very large-scale dimensions; we present a new algorithm that, combining a Decomposition strategy with a Truncated Newton approach, allows to solve this type of problems efficiently. In particular we applied the algorithm to the solution of problems arising in the training of Support Vector Machines, whose dual formulation is characterized by this particular structure.

Joint work with G. Liuzzi, S. Lucidi and F. Rinaldi
On Duality for Mathematical Programs with Vanishing Constraints

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In this paper, we formulate and study Wolf and Mond-Weir type dual models for a difficult class of optimization problems known as the mathematical programs with vanishing constraints. We establish the weak, strong, converse, restricted converse and strict converse duality results under the assumptions of convexity and strict convexity between the primal mathematical program with vanishing constraints and the corresponding Wolf type dual. We also derive the weak, strong, converse, restricted converse and strict converse duality results between the primal mathematical program with vanishing constraints and the corresponding Mond-Weir type dual under the assumptions of pseudoconvex, strict pseudoconvex and quasiconvex functions.

Global Optimization of Uniform Coverage Problems

with Modified Remez Algorithm

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The homogeneous deposition of material on complex surfaces, e.g., powder coating, paint spraying, fibre lay-down etc., is a well-known task in industrial production. This talk introduces a mathematical model for a typical process class of such uniform coverage problems motivated by the spunbond process. The process class is characterised as follows: assume that a bar rotates around a centre with uniform speed and specified constant mass distribution along the bar. Simultaneously, the centre of the bar is moved in some direction along the surface with constant speed. By the superposition of rotational and translational movements, a pattern is generated. In optimisation, it is natural to aim for a modification of mass distribution in relation to rotational and translational speed such that the surface is covered as homogeneously as possible.

We show that uniform coverage problems can be reformulated as linear semi-infinite programs. As a linear semi-infinite program, coverage patterns can be optimised by means of a modified version of Remez algorithm. It finds reliably global solutions in appropriate time.

The talk is organised as follows: After an introduction of the problem and a description of the mathematical model in the first part, an optimisation program is introduced in the second part. In particular, the application of Remez algorithm is discussed. In the last part, numerical results illustrate the performance of this approach. Remez solutions are compared to analytical solutions from an approximate mode.

Joint work with K. H. Küfer
On a Modified Variational Iteration Method for the Analytical Solution of Korteweg-de-Vries Equation

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In this paper, a new analytical technique known as Modified Variational Iteration Method (MVIM) for the solution of Korteweg-de-Vries equation is presented. Numerical examples are tested to illustrate the efficiency, reliability and pertinent feature of the proposed method.

Joint work with F. O. Akinpelu and A. W. Gbolagade

Global Black-Box Optimization in Trading of Securities

STEFANIA RENZI

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Given a trading strategy, the role of optimization consists in finding the best conditions to start a trading maximizing the profit. In this general scenario, the strategy is trained on a chosen batch of data (training set) and applied on the next batch of data (trading set). Given a strategy, there are different issues to deal with, to obtain the best performances from the optimization. First of all, among all the parameters that define the strategy, it is important to identify and select the most relevant ones that become the optimization problem variables. In this way the problem complexity is reduced.

Once the variables are chosen, the focus is on the time period used for the training and the trading sets. Accordingly, for any parameter, a proper box constraint is fixed. A reliable global optimization method must be chosen and then an efficient algorithm based on that method has to be implemented. Characteristic of the problem is that the objective function is not defined in closed form but through an algorithm, the problem lies within the framework of black-box optimization.

This study is aimed to highlight possible relationships between the behavior of the particular security data set and the performance of the global optimization techniques.

Joint work with A. De Santis, U. Dellepiane and S. Lucidi
Time-Optimal Feedback Control for Nonlinear Pendulum-Like Systems

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We consider a nonlinear controlled dynamical system \( \ddot{\varphi} + \varepsilon(\varphi) = u \) that is a generalization of the well-known equation of the pendulum for which \( \varepsilon(\varphi) = \sin \varphi \). Here, \( u(t) \) is a control bounded by the constraint \( |u(t)| \leq k \). The function \( \varepsilon(\varphi) \) is periodic in coordinate \( \varphi \) with a period of \( 2\theta \) and continuously differentiable for all \( \varphi \). Without loss of generality, we assume \( |\varepsilon(\varphi)| \leq 1 \). The initial state \( \{\varphi(0), \dot{\varphi}(0)\} \) of the system is arbitrary, and the terminal state at \( t = T \) is prescribed: \( \{\varphi(T) = \varphi_T + 2\pi n, \dot{\varphi}(T) = 0\} \). Here, \( \varphi_T \in [0, 2\theta] \) is given, and \( n \) is an arbitrary integer. The terminal condition takes into account the periodicity of \( \varepsilon(\varphi) \).

The time \( T \) is to be minimized with respect to admissible control \( u(t) \).

The time-optimal control problem is treated by means of Pontryagin’s maximum principle. Certain numerical results for the case of the pendulum \( (\varepsilon(\varphi) = \sin \varphi) \) are presented for a wide range of parameter \( k \) both for the lower and upper terminal equilibrium states \( (\varphi_T = 2\pi n, \varphi_T = \pi + 2\pi n) \). The feedback optimal control \( u(\varphi, \dot{\varphi}) \) is completely determined by “switching curves” and “dispersal curves” in the plane \( \varphi, \dot{\varphi} \), which bound the domains where \( u = +k \) and \( u = -k \). The control pattern is periodic with respect to \( \varphi \).

The switching curves consist of the points where the control \( u \) changes its sign along an optimal trajectory, whereas the dispersal curves consist of the points where the optimal control can be equal to either \( +k \) or \( -k \), and two optimal trajectories originating at each of these points reach terminal state (for identical or distinct \( n \)) in the same time.

For more general pendulum-like systems, we find explicit estimates \( k \geq \tilde{k}(\theta) > 1 \) on the control constraints, for which the number of switchings is not greater than one for any initial conditions, i.e., the feedback control has the simplest pattern in the phase plane. For the estimated interval of the control constraints, we analyze the structure of the time-optimal feedback control.

Joint work with F. L. Chernousko

DIRECT-type Algorithms for Derivative-Free Constrained Global Optimization

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In the field of global optimization many efforts have been devoted to globally solving bound constrained optimization problems without using the derivatives of \( f \). In this talk we consider global optimization problems where both bound and general nonlinear constraints are present. This is of great practical importance in many real-world applications. To solve this problem we propose the combined use of a DIRECT-type algorithm with derivative-free local minimizations of a nonsmooth exact penalty function. In particular, we define a new DIRECT-type strategy to explore the search space by explicitly
taking into account the two-fold nature of the optimization problems, i.e. the global optimization of both the objective function and of a feasibility measure. We report an extensive experimentation on hard test problems to show viability of the approach.

Joint work with G. Di Pillo, G. Liuzzi, S. Lucidi and V. Piccialli

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**A Krylov based Class of Approximate Inverse Preconditioners for Large Indefinite Linear Systems (AINVK)**

**Massimo Roma**

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We propose a class of preconditioners for symmetric linear systems arising from numerical analysis and nonconvex optimization frameworks. Our preconditioners are specifically suited for large indefinite linear systems and may be obtained as by-product of Krylov-subspace solvers, as well as by applying L-BFGS updates. Moreover, our proposal is also suited for the solution of a sequence of linear systems, say $Ax = b_i$ or $A_i x = b_i$, where respectively the right-hand side changes or the system matrix slightly changes, too. Each preconditioner in our class is identified by setting the values of a pair of parameters and a scaling matrix, which are user-dependent, and may be chosen according to the structure of the problem in hand. We provide theoretical properties of our preconditioners, discussing the relation with the proposals in [1,2]. In particular, we show that our preconditioners both shift some eigenvalues of the indefinite system matrix to $\pm 1$, and are able to control the condition number of the preconditioned matrix. We study some structural properties of our class of preconditioners, and report the results on a comparative numerical experience with LMP preconditioners [1]. The experience is carried on first considering some relevant linear systems proposed in the literature. Then, we embed our preconditioners within a linesearch-based truncated Newton method, where sequences of linear systems (namely Newton’s equations), are required to be solved. We perform an extensive numerical testing over the entire large scale unconstrained optimization test set of CUTEr collection, confirming the efficiency of our proposal.

**References**


Joint work with G. Fasano
A New Hybrid Interior-Point Algorithm
for Quasi-Variational Inequalities

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We present an hybrid interior-point algorithm for finite-dimensional quasi-variational inequalities (QVIs). This method combines the robustness of the potential reduction algorithm [3] and the local quadratic convergence rate of the LP-Newton method [2]. We provide local and global convergence properties of this method along with a comprehensive description of the algorithm, including matrix corrections for singular system cases and crash methods. Heuristics are also considered that allow faster performance. This method has been implemented in C++, which we demonstrate on QVILIB [4], a large test problem library of QVIs. An evaluation is made of various potential reduction line-search and linear solver options, and a comparison is provided with PATH [1], the state-of-the-art code for complementarity problems.

References


Joint work with F. Facchinei and C. Kanzow

An Optimization Scheme for the Computation of Mean Field Games Equilibria

JULIEN SALOMON

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In this talk, we consider a mean field games model for the choice of technologies (with externalities and economy of scale) and present a numerical method to solve the corresponding optimization problem. This procedure is designed by adapting the so-called monotonic algorithms that were initially introduced in the field of quantum chemistry. We conclude with some numerical results. We show in particular the existence of multiple equilibria describing the possibility of a technological transition. Joint work with A. Lachapelle and G. Turinici
Consistency of General Variational Learning Schemes

Saverio Salzo

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In Machine Learning, a common learning scheme is obtained by the exact minimization of the Tikhonov regularized empirical risk. In this work a theoretical study of general variational learning schemes is presented. The proposed approach employs a regularization function satisfying only a suitable uniform convexity property and allows the corresponding minimization to be inexact. In this framework, weak and strong consistency theorems are provided, which show the convergence of the scheme as the regularization parameter and the precision error tend to zero. The role played by the modulus of convexity in the learning rates is also highlighted. As an application, we show that regularizing the empirical risk with the power of the $p$-norm, $1 < p < 2$, gives a consistent learning scheme.

Joint work with P. L. Combettes and S. Villa

Extremal Problems Under Various Invariance Assumptions

Alberto Seeger

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We establish a localization result for the solutions to a broad class of extremal problems involving invariant sets and invariant functions. The concept of invariance is relative to a special collection of linear endomorphisms on some Euclidean space. As a particular instance, we discuss the case of extremal problems involving orthogonally invariant matrix functions.

Approximately Star-Shaped Functions and Approximate Vector Variational Inequalities with Support Functions

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In 1998, Jofre et al. [1] introduced $\epsilon$–convex functions as a generalization of convex functions. It was found that these functions are very useful in approximate calculus and a lot of theory about the convex functions can be extended for $\epsilon$–convex functions. Using $\epsilon$–convex functions as a basic tool, Ngai et al. [2] defined a new class of generalized convex functions over a Banach space, called approximate convex functions, which not only contains the class of convex functions, but also the class of continuously differentiable functions. Later, Daniilidis and Georgiev [3] characterized locally Lipschitz approximately convex functions using the Clarke subdifferentials.

Let $X$ be a normed linear space and let $\epsilon > 0$ be given. A function $g : X \to R \cup \{+\infty\}$ is said to be $\epsilon$–convex on a convex subset $K$ of $X$, iff for every $x, y \in K$ and $\lambda \in [0, 1]$, one has

$$g(\lambda x + (1 - \lambda)y) \leq \lambda g(x) + (1 - \lambda)g(y) + \epsilon \lambda(1 - \lambda)\|x - y\|.$$
The function $g$ is said to be *approximate convex* at $\bar{x} \in X$, iff $g(\bar{x})$ is finite, and for any $\epsilon > 0$, there exists $\delta > 0$ such that, $g$ is $\epsilon-$convex on $B(\bar{x};\delta)$. The *Clarke directional derivative* of $g$ at $\bar{x} \in domg := \{x \in X : g(x) < +\infty\}$ in a direction $u \in X$ is given by

$$f^o(\bar{x};u) := \limsup_{(x,t) \rightarrow (\bar{x},0^+)} \frac{f(x+tu)-f(x)}{t}$$

and the *Clarke subdifferential* of $g$ at $\bar{x}$ is the subset $\partial^o g(\bar{x})$ of $X^*$ given by

$$\partial^o g(\bar{x}) := \{ \bar{x}^* \in X^* : f^o(\bar{x};u) \geq \langle \bar{x}^*, u \rangle, \forall u \in X \}$$.

We consider the following vector optimization problem:

(VOP) $\min (f_1(x) + s(x|C_1), \ldots, f_m(x) + s(x|C_m))$ subject to $x \in X$, where $f_i : X \rightarrow R \cup \{+\infty\}$ is locally Lipschitz on $X$ and approximately convex at $\bar{x} \in X$ for all $i \in M := \{1, \ldots, m\}$. Also, $C_i$ is a compact convex subset of $X$ and

$$s(x|C_i) := \max \{ \langle x, y \rangle : y \in C_i \}, \forall i \in M.$$  

We give the concept of approximate efficient solutions introduced by Mishra and Laha (4) in the framework of the VOP with support functions. A vector $\bar{x} \in X$ is called an *approximate efficient solution* of the VOP, iff for any $\epsilon > 0$, there does not exist $\delta > 0$ such that, for all $i \in M$ and $x \in B(\bar{x};\delta)$, one has

$$[f_i(x) + s(x|C_i)] - [f_i(\bar{x}) + s(\bar{x}|C_i)] \leq \epsilon \|x - \bar{x}\|,$$

with strict inequality for at least one $i \in M$. We generalize the approximate vector variational inequalities introduced in (4) as follows:

(ASVVI)$_1$ To find $\bar{x} \in X$ such that, for any $\epsilon > 0$, there does not exist $\delta > 0$ such that, for all $i \in M$, $x^*_i \in \partial^o f_i(\bar{x})$, $w^*_i \in \partial^o s(\bar{x}|C_i)$ and $x \in B(\bar{x};\delta)$, one has

$$\langle x^*_i + w_i^*, x - \bar{x} \rangle \leq \epsilon \|x - \bar{x}\|,$$

with strict inequality for at least one $i \in M$.

(AMVVI)$_1$ To find $\bar{x} \in X$ such that, for any $\epsilon > 0$, there does not exist $\delta > 0$ such that, for all $i \in M$, $x, x^*_i \in \partial^o f_i(x)$ and $w^*_i \in \partial^o s(x|C_i)$, one has

$$\langle x^*_i + w_i^*, \bar{x} - x \rangle \leq -\epsilon \|\bar{x} - x\|,$$

with strict inequality for at least one $i \in M$.

In this talk, we consider the VOP involving approximately convex functions and discuss the relationships of the approximate efficient solutions of the VOP with the solutions of the (ASVVI)$_1$ and (AMVVI)$_1$. We also consider the corresponding weak versions of the approximate vector variational inequalities and establish various results for approximate weak efficient solutions.

References


Joint work with S. K. Mishra and V. Laha
New Methods for Solving Nonconvex Optimal Control Problems

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As known, the most of real-life optimization problems turn out to be both nonconvex and dynamic, what provides a huge of difficulties and singularities in studying and moreover in a numeric search for a global solution to the problems.

We consider optimal control (OC) problems with quadratic functionals defined by matrices which are indefinite, otherwise the problems with integral and terminal (d.c.) functionals, representable as a difference of two convex functionals (Bolza problems). For this class of optimal control problems we propose, first, special local search methods, consisting in consecutive solving the linearized (w.r.t. the basic nonconvexities at a current iteration) problems, and a study of its convergence [5]. Second, for the OC problems under scrutiny Global Optimality Conditions (GOC) developed, from which in particular the Pontryagin maximum principle follows [4,8]. On the base of these GOC a family of Global Search Methods (GSM) have been developed and its convergence was investigated [6,7].

Besides a family of Local Search methods special for each kind of nonconvexity was proposed and substantiated, and after that incorporated into Global Search Procedures [3].

Further the number of special nonconvex OC test problems has been generated by the procedure the idea of which belong to L.N.Vicente and P.H.Calamai [1,2].

On this large field of benchmarks problems some of that are of rather high dimension (20 in state and 20 in control) it was conducted a large number of computational experiments which witnessed on the attractive abilities and the promising effectiveness of the developed approach.

References


The Halley class of methods is a class of higher order methods. Methods in this class are under suitable assumptions locally convergent and have a third order rate of convergence. The Halley class contains well known methods, such as Chebyshev, Halley and super-Halley methods. The Halley class can be written as a two-stage methods where the new iterate is updated by a combination of the current point and two directions; the Newton direction and a correction direction using the third derivative.

Curvilinear search is a global strategy that searches along a curve defined by two directions using backtracking. The two most used directions in unconstrained optimization are defined by a descent direction and a direction of negative curvature. The descent direction is utilizing a modified factorization of the Hessian matrix to guarantee descent and stability. Rarely, a curvilinear search uses a direction based on third order derivatives.

In this paper, we present an algorithmic framework for solving unconstrained optimization problems. It is based on introducing the curvilinear search as a combination of Newton’s direction and a direction uses tensor/higher order derivatives of the objective function. We show under standard assumptions that the Halley class combined with a curvilinear search is globally convergent. In addition, limit points of the sequence of iterates will be stationary points. Under suitable choices of the directions the Hessian matrix at the limit points will be positive semidefinite. Moreover, if the Hessian at the limit-point is positive definite then the sequence of iterates converges with Q-cubic rate of convergence. We also provide numerical experiments on test problems from the MINPACK and CUTE collections which illustrate the theoretical findings.

Joint work with T. Steihaug

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Sum rate maximization problem is always of great interests in the field of wireless communications. For MIMO relay networks, we propose a new approach to approximate sum rate maximization, and prove it is a lower bound of achievable sum rate. To solve the nonlinear nonconvex optimization problem, we first change the fraction function into a non-fraction function in the objective function, and show that the optimization problems share the same stationary points. By applying the alternating minimization method, we decompose the complex problem into several subproblems that are easier handled with. Moreover, we prove that the proposed models always lead to rank one solutions. From practical demand, we also add orthogonal constraints and solve the corresponding problem.

Joint work with E. Jorswieck
Convergence of a Family of Discrete Neumann Boundary Elliptic Optimal Control Problems with Respect to a Parameter

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We consider a bounded domain Ω in \( \mathbb{R}^n \) whose regular boundary \( \partial \Omega \) consists of the union of two disjoint portions \( \Gamma_1 \) and \( \Gamma_2 \) with \( \text{meas}(\Gamma_1) > 0 \) and \( \text{meas}(\Gamma_2) > 0 \).

The convergence of a family of continuous Neumann type boundary optimal control problems \( P_\alpha \), governed by elliptic variational equalities, when the parameter \( \alpha \) of the family (the heat transfer coefficient on the portion of the boundary \( \Gamma_1 \)) goes to infinity was studied in Gariboldi-Tarzia, Adv. Diff. Eq. Control Processes, 1 (2008), 113-132. The control variable is the heat flux on the portion of the boundary \( \Gamma_2 \). It has been proved that the optimal control, and their corresponding system and adjoint states are strongly convergent, in adequate functional spaces, to the optimal control, and the system and adjoint states of another Neumann type boundary optimal control problem \( P \) governed also by another elliptic variational equality with a different boundary condition on the portion of the boundary \( \Gamma_1 \).

We consider the discrete approximations \( P_{h\alpha} \) and \( P_h \) of the continuous optimal control problems \( P_\alpha \) and \( P \) respectively, for each \( h > 0 \) and for each \( \alpha > 0 \), through the finite element method with Lagrange’s triangles of type 1 with parameter \( h \) (the longest side of the triangles). We also discrete the elliptic variational equalities which define the systems and their adjoint state, and the corresponding cost functional of the Neumann boundary optimal control problems \( P_\alpha \) and \( P \).

The goal of this paper is to study the convergence of this family of discrete Neumann boundary elliptic optimal control problems \( P_{h\alpha} \) when the parameter \( \alpha \) goes to infinity. We prove the convergence of the discrete optimal controls, the discrete system and adjoint states of the family \( P_{h\alpha} \) to the corresponding to the discrete Neumann optimal control problem \( P_h \) when \( \alpha \to +\infty \), for each \( h > 0 \), in adequate functional spaces.

Optimizing FSSP Algorithms for One-Bit-Communication Cellular Automata

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In recent years, interest in cellular automata (CA) has been increasing in the field of modeling real phenomena that occur in biology, chemistry, ecology, economy, geology, mechanical engineering, medicine, physics, sociology, and public transportation. Cellular automata are considered to provide a good model of complex systems in which an infinite array of finite state machines (cells) updates itself in a synchronous manner according to a uniform local rule. In the present paper, we study a firing squad synchronization problem (FSSP) on a special subclass of cellular automata: one-bit inter-cell communication cellular automaton.

The O(1)-bit communication model is a conventional CA in which the number of communication bits exchanged in one step between neighboring cells is assumed to be O(1) bits. However, such bit
information exchanged between inter-cells is hidden behind the definition of conventional automata-theoretic finite state descriptions. On the other hand, the 1-bit inter-cell communication model studied in the present paper is a new subclass of CAs, in which inter-cell communication is restricted to 1-bit communication. We refer to this model as the 1-bit CA and denote the model as CA$^{1\text{-bit}}$. The number of internal states of the CA$^{1\text{-bit}}$ is assumed to be finite as in a usual sense. The next state of each cell is determined based on the present state of the cell and two binary 1-bit inputs from its left and right neighbor cells. Thus, the CA$^{1\text{-bit}}$ is one of the weakest and simplest models among the variants of the CAs.

A main question in this paper is whether the CA$^{1\text{-bit}}$ can solve any FSSP problems solved by conventional cellular automata without any overhead in time complexities. We also study an integer sequence generation problem, a French flag problem, and an early bird problem, all of which are classical, fundamental problems that have been studied extensively on O(1)-bit communication models of cellular automata.

References


Generalized Vector Variational Like Inequalities and Nonsmooth Vector Optimization Using Limiting Subdifferential

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In this paper, two generalized Minty vector variational like inequalities, defined by means of Moreau-Dukhovich limiting subdifferentials are considered. We investigate the relations between the solutions of these vector variational like inequalities and vector optimization problems for non-differentiable higher order generalized invex functions in Asplund space setting by using the properties of limiting subdifferentials.
Optimal Management of Medium-Voltage AC Networks with Distributed Generation and Storage Devices

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A medium-voltage AC network with distributed generation and storage devices is considered for which set points are assigned in each time period of a given time horizon. A set point in a time period is defined by modules and phases of voltages in all nodes, active and reactive powers, on load tap changer and variable loads. When some parameters vary, in order to restore feasibility new set points need to be determined so as to minimize the variations with respect to the initial ones. This can be done by minimizing distributors redispatching costs, which are modelled by means of binary variables, while satisfying service security requirements and ensuring service quality, which are represented by nonlinear constraints, such as the nodal balance of active and reactive power and the current transits on lines and transformers for security. Storage devices are modeled by means of constraints that relate adjacent time periods. A two-step solution procedure is proposed, which is based on decoupling active and reactive variables: in the first step a MILP model determines the active power production and the use of storage devices that minimize redispatching costs over all time periods in the time horizon; in the second step, given the optimal active power production computed in the first step, reactive variables in each time period are computed by solving a nonlinear programming model.

Joint work with A. Bosisio, D. Moneta and S. Zigrino

A Proximal Gauss-Newton Method

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An extension of the Gauss-Newton algorithm is proposed to find local minimizers of penalized nonlinear least squares problems, under generalized Lipschitz assumptions. Convergence results of local type are obtained, as well as an estimate of the radius of the convergence ball. Some applications for solving constrained nonlinear equations are discussed and the numerical performance of the method is assessed on some significant test problems.

Joint work with S. Salzo
A Mean Field Game Approach for Evacuation Situations

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In this talk we present a mean field game model for evacuation situations, where people want to leave a domain $\Omega$ as fast as possible. To do so every pedestrian tries to minimize a certain cost functional on the microscopic level, which leads to a nonlinear optimization problem in the macroscopic setup. We discuss the behavior of the proposed optimal control approach and the relation to existing models in the literature. Furthermore we illustrate the behavior with different numerical simulations.

Joint work with M. Burger, M. Di Francesco and P.A. Markowich